Entanglement generation between two spin-*s* magnetic impurities in a solid via electron scattering

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Abstract

We present a scheme for generating entanglement between two magnetic impurities in a solid-state system via electron scattering. The scheme applies to impurities of arbitrary quantum spin number. We show that resonance conditions yield generation of a maximally entangled state of the impurities' spins, regardless of the value of the electron−impurity coupling constant and the impurity spin quantum number. The mechanism behind the scheme is explained in terms of resonance-induced selection rules.
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The key role played by entanglement in many protocols for quantum information processing (QIP) is now firmly grounded [1]. The necessity of making such a powerful resource easily available has thus boosted the quest for efficient techniques able to generate entangled states. Although important progresses have been performed in various physical settings, the reliable generation of usable entangled remains, in general, a rather demanding task. In particular, the creation of bipartite entanglement between spins in solid-state nanostructures is an important challenge that attracts rapidly growing interest due to the promises for scalable QIP held by solid-state systems [1,2].

Over the last few years, several schemes have been proposed based on the direct interaction between two parties belonging to a solid-state scenario, both for adjacent stationary spins [3,4] and mobile ones [5−10]. In this respect, strategies able to entangle stationary spins (or any sort of stationary qubits) separated by a distance longer than their interaction-length have been put forward. The importance of these schemes is understood by observing that a viable method to scale the performances of a quantum computer (and efficiently protect it from some environmental effects) is given by the use of linked remote quantum registers [11,12] (if stationary qubits belonging to distinct quantum registers are entangled, they can be effectively considered as parts of the same quantum computer). The first proposals in this area required a high degree of control over the interaction times within a system. Indeed, they relied on the shuttling of ions/spins over a given distance, combined with the application of precisely timed gates between shuttled and stationary qubits [11,12]. Obviously, they are rather impractical.

Recently, it was shown that it is possible to entangle two stationary spins outside their interaction-length under situations of *reduced control*. This can be accomplished via multiple scattering of a conduction electron between two magnetic impurities embedded in a solid such as a semiconductor quantum wire [13,14]. Seminal works proposing such an idea essentially harnessed conservation of the total spin quantum number and relied on approximated approaches. Indeed, either

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they fully neglected electron back scattering [14] or took into account only a finite number of multiple reflections [13]. More recently, some of us have tackled the very same problem using a quantum-waveguide theory approach, deriving the transfer matrix at all orders in the electron−impurity coupling constant and in the distance between the two impurities [15,16]. This has shed some light onto the effect of resonance conditions in the establishment of entangled states of the impurities. Indeed, under resonance conditions, the equal probabilities to find an electron at the two impurities' locations provide an additional Hamiltonian symmetry, namely the conservation of the squared total spin of the scattering centers. This resonance-induced selection rule allows to design a scheme that ensures the generation of maximally entangled states, regardless of the strength of the electron−impurity coupling [15,16]. This scheme has been shown to be rather robust against static disorder [17]. A parametric study of the very same scheme was presented in Ref. [18], while a different proposal addressed the entanglement generation via electronic mediator in a circular array of quantum dots [19]. Noticeably, the same resonance-induced symmetry is responsible for other interesting quantum transport phenomena, such as entanglement-controlled transmittivity [15] and entanglement-induced survival of the Aharonov−Bohm oscillations in a mesoscopic ring with magnetic impurities [20].

To the best of our knowledge, only the case of spin-1/2 magnetic impurities has been addressed [13−19]. Indeed, one qubit can be exactly encoded in the spin degree of freedom of a spin-1/2 particle. Actual magnetic impurities, however, may have larger spin. A common case is encountered in Mn-doped semiconductors where the Mn atom has spin 5/2. In this paper, we propose a strategy to generalize the scheme for entanglement generation between two spin-1/2 magnetic impurities [15,16] to the case of two spin-s impurities. In particular, we focus on resonance conditions with the aim of exploring how they affect entanglement generation in the case of magnetic impurities with spin higher than 1/2.

We consider a quasi-one-dimensional (1D) wire where a conduction electron scatters two spin-s magnetic impurities, labeled 1 and 2 and embedded at $x = 0$ and $x = x_0$, respectively. The system is sketched in Fig. 1. Provided the electron's coherence-length exceeds the impurity distance x_0 and the electron energy is low enough that only the first subband
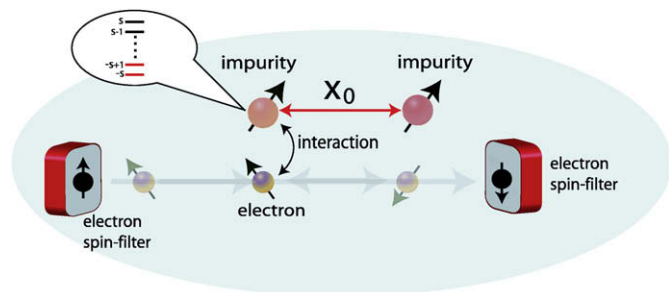

Fig. 1. Scheme for the entanglement between two magnetic impurities in a solid by means of electron scattering. The electron flies along a 1D wire and scatters off each of the two impurities, effectively entangling them.

is occupied, the system is described by the following Hamiltonian:

$$H = \frac{p^2}{2m^*} + J\boldsymbol{\sigma}\cdot\mathbf{S}_1\delta(x) + J\boldsymbol{\sigma}\cdot\mathbf{S}_2\delta(x - x_0), \tag{1}$$

where $p = -i\hbar\nabla$, m^* and $\boldsymbol{\sigma}$ are the electron momentum operator, effective mass and spin-1/2 operator, respectively, $\mathbf{S}_i$ ($i = 1, 2$) is the spin-s operator of the i-th impurity and J is the exchange spin−spin coupling constant between the electron and each impurity. We indicate with $\mathbf{S} = \boldsymbol{\sigma} + \mathbf{S}_1 + \mathbf{S}_2$, $\mathbf{S}_{ei} = \boldsymbol{\sigma} + \mathbf{S}_i$ and $\mathbf{S}_{12} = \mathbf{S}_1 + \mathbf{S}_2$ the total spin of the system, the total spin of the electron plus i-th impurity and the total spin of the two impurities, respectively. Note that $\mathbf{S}_{ei}^2$ and $\mathbf{S}_{12}^2$ (with associated quantum numbers s_{ei} and s_{12}) do not commute, in general, with the Hamiltonian (1). On the other hand, $\mathbf{S}^2$ and S_z (with associated quantum numbers S and m) are conserved quantities. Therefore, H can be block-diagonalized, each block corresponding to a pair $\{S, m\}$.

Due to contact electron−impurity exchange interaction, spin-flip events may take place when the electron elastically scatters off an impurity. As a result, the overall spin state of the system is generally different from the initial one. Consider an electron incoming from the left lead with wavevector k and assume that its spin can be prepared at the input of the wire in the state $|\uparrow\rangle_e$ and measured at its output in $|\downarrow\rangle_e$ (see Fig. 1). This task could be achieved through ferromagnetic contacts, acting as electron spin-filters, placed at the source and drain of the wire like in typical spintronic devices [21]. Moreover, let us consider a spin-polarized electron incoming from the left lead with wavevector k and spin $|\uparrow\rangle_e$ with the impurities initially prepared in $|-s, -s\rangle_{12}$. According to the conservation of S_z, multiple electron−impurity scattering events change the initial overall spin state $|\uparrow, -s, -s\rangle$ into the *transmitted* state:

$$|\chi_t\rangle = A|\uparrow, -s, -s\rangle + B|\downarrow, -s+1, -s\rangle + C|\downarrow, -s, -s+1\rangle, \tag{2}$$

where A, B and C are transmission probability amplitudes. The form of this state implies that when the electronic spin is post-selected by spin-filter in $|\downarrow\rangle$ after the transmission (cfr. Fig. 1), the impurities are projected onto

$$|\chi_{t,\downarrow}\rangle_{12} = B|-s+1, -s\rangle_{12} + C|-s, -s+1\rangle_{12}, \tag{3}$$

up to a normalization factor. Such event occurs with probability $T_\downarrow = |B|^2 + |C|^2$, $T_\downarrow$ being a spin-polarized electron transmission coefficient. State (3) is, in general, an entangled state of the two impurities.

Eqs. (2) and (3) represent a generalization of the scheme proposed in Refs. [13,15,16]. This suggests that, even though each impurity has a $(2s + 1)$-dimensional Hilbert space, their spin states $|-s\rangle$ and $|-s + 1\rangle$ can be used to encode one qubit (see Fig. 1) since selection rules prevent spin states $|-s + 2\rangle, ..., |s\rangle$ to be involved (a similar idea has been presented in Ref. [22] with a different pair of states). The coefficients B and C appearing in Eq. (3) can be calculated as functions of s, kx_0 and $\rho(E)J$ through a direct generalization

of the technique used in Refs. [15,16] [$E = (\hbar^2 k^2/2m^*)$ and $\rho(E)$ are the electron energy and the density of states per unit length of the wire, respectively]. The calculation utilizes the probability amplitudes $t_{s_{e2}}^{(s'_{e2},S)}$ that an electron, incoming from the left with wavevector k and with the system initially in the spin state $|s'_{e2}; S, m\rangle$, is transmitted (after multiple scattering) with the spin state changed into $|s_{e2}; s, m\rangle$ $(s_{e2}, s'_{e2} = s - 1/2, s + 1/2)$ [15,16]. Although there is a proliferation of such coefficients with the growing of the impurity spin number s, only those with $S = 2s - 1/2$ and $S = 2s + 1/2$ need to be calculated. This is due to the fact that each spin state appearing in the right-hand side of Eq. (2) does not belong to subspaces having $S < 2s - 1/2$. These coefficients are computed solving the Schrödinger equation and imposing the matching of the wavefunction at $x = 0$ and $x = x_0$.

Clearly, the entanglement-generation process is optimized when the scheme yields a maximally entangled state, *i.e.* when the von Neumann entropy of state (3) $E = -\text{Tr}(\rho_1 \lg_2 \rho_1)$ [1] becomes 1 (ρ_1 is the density matrix describing the state of impurity 1). In the case of two impurities of spin $s = 1/2$, this goal is achievable in the regime of resonance conditions $kx_0 = n\pi$ (with n an integer) [15,16,18]. In Fig. 2 we plot the spin-polarized transmission coefficient $T_\downarrow$ [*i.e.* the probability to generate state (3)] together with the von Neumann entropy E of state (3) as functions of $\rho(E)J$ and at resonance conditions for different values of the impurity spin number s (the case $s = 1/2$ is also reported for a comparison). The results of Refs. [15,16] are recovered in the case $s = 1/2$, with $E = 1$ for any strength of $\rho(E)J$ and with $T_\downarrow$ showing a maximum at $\rho(E)J \simeq 1$. We can see that $T_\downarrow$ vanishes at small and large values of $\rho(E)J$. This is due to the fact that, in such conditions, electron–impurity spin-flipping, *i.e.* the mechanism responsible for entanglement generation, is negligible [23]. An analogous qualitative behavior is exhibited by higher-spin impurities. The maximum value of $T_\downarrow$ diminishes and is shifted towards smaller values of $\rho(E)J$, as s increases. Remarkably, the resonance conditions yield $E = 1$ regardless of $\rho(E)J$ and for any spin s. The ability of the scheme to generate a maximally entangled state thus holds for high-spin
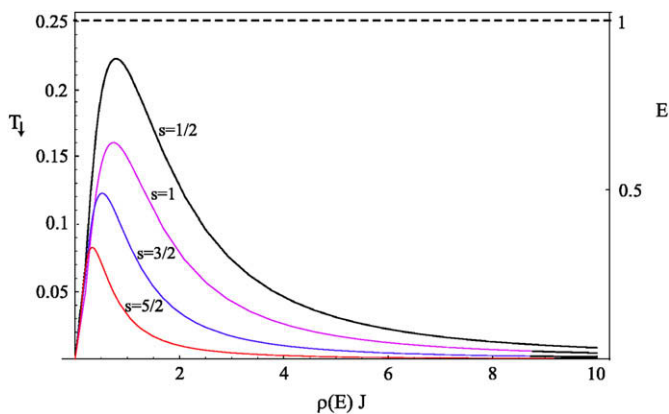

Fig. 2. Spin-polarized electron transmission coefficient $T_\downarrow$ (solid line) and von Neumann entropy E (dashed line) at $kx_0 = n\pi$ as a function of $\rho(E)J$ and for the cases $s = 1/2, 1, 3/2, 5/2$. The electron arrives prepared in $|\uparrow\rangle_e$ and the impurities are initially in $|-s, -s\rangle_{12}$.

impurities as well. The mechanism behind this result is explained as follows. As pointed out previously, the squared total spin of the impurities $\mathbf{S}_{12}^2$ is in general not conserved due to the effects of the Dirac functions $\delta(x)$ and $\delta(x - x_0)$ in (1). However, when $kx_0 = n\pi$ the effective representations $\delta_k(x)$ and $\delta_k(x - x_0)$ of these two electronic orbital operators coincide (the electron being found at $x = 0$ and $x = x_0$ with equal probabilities) and $\mathbf{S}_{12}^2$ turns out to be an additional constant of motion. Resonance conditions therefore allow conservation of S_z and $\mathbf{S}_{12}^2$ to hold simultaneously. In our scheme, the initial spin state $|\uparrow\rangle_e|-s, -s\rangle_{12}$ is an eigenstate of S_z and $\mathbf{S}_{12}^2$ with quantum numbers $m = 1/2 - 2s$ and $s_{12} = 2s$, respectively. Regardless of the spin number s, there is only another state having the same quantum numbers. This is $|\downarrow\rangle_e|s, s, s_{12} = 2s, m_{12} = -2s + 1\rangle_{12}$ (where the standard notation used in the coupling of angular momenta has been used and where m_{12} is the quantum number of $S_{12,z}$). It follows that, by ensuring resonance conditions, multiple electron–impurity scattering can either leave the initial spin state $|\uparrow\rangle_e|-s, -s\rangle_{12}$ unchanged or changes it onto $|\downarrow\rangle_e|s, s, 2s, -2s + 1\rangle_{12}$. Post-selection of state $|\downarrow\rangle_e$ of the transmitted electron thus projects the impurities onto state $|s, s, 2s, -2s + 1\rangle_{12}$. Using the Clebsch–Gordan coefficients, such state turns out to be a linear superposition of $|-s + 1, -s\rangle_{12}$ and $|-s, -s + 1\rangle_{12}$ according to

$$|s, s, 2s, -2s + 1\rangle_{12} = \frac{1}{\sqrt{2}}(|-s + 1, -s\rangle_{12} + |-s, -s + 1\rangle_{12}).$$

$$(4)$$

State (4) has $E = 1$ and is therefore a maximally entangled state regardless of the value of s. Indeed, at $kx_0 = n\pi$, we find that $B = C = 1/\sqrt{2}$ up to an irrelevant global phase factor and regardless of the value of $\rho(E)J$ and s.

In conclusions, in this paper we have extended to arbitrary quantum spin number a previously proposed scheme for the generation of entangled states of two magnetic impurities valid for $s = 1/2$. Entanglement is established via a conduction electron that undergoes multiple scattering with simultaneous spin-flip and whose spin state is finally post-selected. We have demonstrated that resonance conditions allow an additional selection rule to hold. This mechanism ensures the generation of a maximally entangled state regardless of the value of the electron–impurity coupling constant and the impurity spin number. It should be mentioned that in general electron transport in the presence of magnetic impurity scattering might be hindered by many-body phenomena such as the Kondo effect [24]. These have been in fact implicitly neglected by the single-electron-approach adopted in this work. Such approximation is expected to be valid in the regime of low electron density. Injection of single electrons could be accomplished by using a single-electron turnstile [25], as suggested e.g. in Refs. [26,27]. We believe that the results presented in this work hold promises for a solid-state implementation of our scheme given that magnetic impurities may actually have spin higher than 1/2. For instance, this is the case of manganese impurities in Mn-doped semiconductors where the Mn atom typically has spin 5/2. Although the probability to generate the maximally entangled state

progressively decreases as s increases, it is still quite significant ($>8\%$) in the case $s = 5/2$. It should be pointed out that our model relies on the assumption that electron–impurity interaction occurs solely via spin–spin exchange coupling. This is not the case when the Mn atom is ionized in semiconductor hosts and therefore an electrostatic electron–impurity coupling is also active. Of course, such difficulty could be overcome provided the strength of the exchange coupling largely exceeds that of the electrostatic one. This regime might, however, be problematic given that the optimal exchange coupling constant maximizing the probability to generate entangled states decreases as s increases (see Fig. 2). However, in a CdTe host, the Mn atom is not ionized due to valence-number matching. Interestingly, in a recent experiment, a single manganese atom was successfully doped in a CdTe quantum dot [28]. CdTe: Mn quantum wires therefore appear as potential candidates for the implementation of our protocol.

Acknowledgements

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