HoloQuantum Network Theory

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The complete general fundamental theory of the dynamical hypergraphs whose 'all' mathematical structures are quantized, *HoloQuantum Network Theory*, is originally defined and formulated based upon a unique system of *nine principles.* HoloQuantum Networks are the quantum states of a (0+1) dimensional purely-information-theoretic quantum many body system, made of a complete set of the distinctively-interacting qubits of the absences-or-presences, which formulate the most complete unitary evolutions of the most general superpositions of the arbitrarily-structured hypergraphs. All the defining interactions and the complete total Hamiltonian of the quantum many body system of HoloQuantum Network Theory are uniquely obtained upon realizing the dynamical hypergraphical well-definedness by all the 'cascade operators', the quantum-hypergraphical isomorphisms, the $U(1)$ symmetry for the global-phase redundancies of the multi-qubit wavefunctions, the minimally broken symmetry of the qubits-equal-treatment, a Wheelerian maximal randomness, and the 'covariant completeness'. By the axiomatic definition and construction of the theory, HoloQuantum Networks are all the time-dependent purely-information-theoretic wavefunctions, and mixed states, of every in-principle-realizable quantum many body system of the arbitrarily-chosen quantum objects and their arbitrarily-chosen quantum relations. Being so, *we propose HoloQuantuam Network Theory as the most-fundamental most-complete form-invariant 'it-from-qubit' theory of 'All Quantum Natures'.*

I. MOTIVES AND METHODOLOGY

Hypergraphs represent *the objects and the relations between them* in the most primitive general abstract way. The original theory which we present in this work formulates the perfection of this representation as *the 'it-from-qubit'* [1] *form-invariant theory of 'all quantum natures'.* That is, all the observables and all the states of all 'physically-possible' quantum many body systems of the objects-and-relations are form-invariantly captured by the so-formulated '*HoloQuantum Network Theory*'.

Mapping time-dependently to the mathematical space of (the maximally oriented-and–weighted) hypergraphs, the sufficiently-refined choices of the vertices represent the arbitrarily-chosen objects, and the sufficiently-refined choices of the hyperlinks represent the arbitrarily-chosen relations between the objects. Mapping so, therefore, the hyperlinks represent all possible physical interactions, classical statistical correlations, quantum entanglements, geometric relations, causality relations, structural or functional relations, emergent relations, the multi-scale relations, and in sum, all possible observable relations. Redefining these same representations as *networks*, we formulate all the time-dependent objects-and-relations states and observables purely information-theoretically.

The objects and their relations can be one by one *either quantum entities or classical entities.* If classical, they are momentarily either absent or present. Being quantum entities, each one object or relation can be in the superposition states of absence and presence.

Moreover, any isolated system of the quantum objects and their quantum relations does form in its entirety a quantum many body system with a unitary evolution. The total Hamiltonian of this unitary dynamics must be determined by all the many-body interactions between all the quantum degrees of freedom, which indeed are, all the quantum objects and all their quantum relations. HoloQuantum Network Theory is formulated in this *framework of quantum many body systems.* Namely, it defines and formulates purely-information-thepretically all the dynamical mathematically-quantized hypergraphs whose defining qubits and their interactions all behave as quantumly as in closed quantum many body systems.

Being its central physics-motive, HoloQuantum Network Theory is 'The Answer' to the following Question.

Question:
Let us assume that quantum physics is the exact law-book of the entire universe or the entire multiverse. Moreover, let us define *'all quantum natures'* to be the entire quantum universe-or-multiverse, as well as all the arbitrarily-chosen algebraically-closed subsystems of her total observables probed in all the jointly-consistent scales, or to be stated equally, all the physically-possible quantum many body systems of quantum objects and their quantum relations. Upon this assumption and with this definition, *what is the quantum many body theory* which most completely, most fundamentally and most compactly formulates all these quantum natures, so that context-independently, both its quantum statics (all the microscopic degrees of freedom) and quantum dynamics (all the many-body interactions and the total Hamiltonian) remain form-invariant = 'covariant'?

We obtain the answer to this question by treating it as an 'equation' and working out its 'solution' constructively. The solution must be the unique construction in which two initially-distinguished faces consistently meet one another, and merge-in to form an indistinguishable unit being *a 'quantum mathematics of all physics'*:

On its *'mathematics face'*, the theory must be the most fundamental, general and complete formulation of dynamical hypergraphs which are all-structurally quantized, 'HoloQuantum Hypergraphs'. Being so, it must formulate most fundamentally and capture most completely all the dynamical mathematically-quantized hypergraphs which are arbitrarily structured, arbitrarily flavored, arbitrarily weighted and arbitrarily oriented. On its *'physics face'*, the same theory must be the most fundamental formulation of the unitarily-evolving total many body system of all the absence-or-presence qubits which code fully-covariantly the most complete time-dependent information of every quantum nature.

Before undertaking the from-first-principle construction of this unique theory, to be accomplished in sections two to five, we first briefly characterize the operational way in which the 'all quantum natures' in the original question is being defined. This precise characterization, serving as a preliminary view, will secure that we will be all clear about the defining physics-context of the resulted quantum mathematics.

First, let us consider the *entire quantum universe*, or to be maximally general as we must, the *entire quantum multiverse*. Physics-wise, this entire quantum universe-or-multiverse is defined as the maximally-large maximally-complete set of $C^{\star}$-algebra-forming quantum observables which are based on the Hilbert space of the total set of independent quantum degrees of freedom. By construction, the evolving global wavefunctions of this initial 'all-inclusive quantum many body system' contain the complete time-dependent information which are decodable by all possible measurements on it.

Second, we now consider all possible operations of *'phenomenological reductions'*. By definition, every such operation corresponds to an arbitrarily-chosen proper subset of that entire set of quantum observables, which are based on the corrsponding Hilbert space of an arbitrarily-chosen proper-or-improper subset of that total set quantum degrees of freedom, are scale-preserved in the sense of the renormalization group flow, and all together form a $C^{\star}$-subalgebra. Now, every so-identified phenomenological reduction, being a likewise-reduced quantum many body system, defines observationally *a 'reduced quantum universe-or-multiverse'*. Clearly enough, for every such reduced universe-or-multiverse, the likewise-constructed evolving global wavefunctions contain the complete time-dependent information which are decodable by all possible measurements on it.

Third, to every such collection of the $C^{\star}$-algebra-forming quantum observables and their base Hilbert spaces, being the complete universe-or-multiverse or each one of the reduced universes-or-multiverses, we apply all possible operations of *'probes rescalings'*, by which one's scale of probing the observables is being re-chosen in all the arbitrarily-chosen ways that are only constrained by all the algebraic and observational consistencies. We so call the resulted quantum many body systems the *'renormalized quantum universes-or-multiverses'*. Now, as before, the likewise-constructed evolving global wavefunctions of every one of these renormalized quantum universe-or-multiverse contains its complete time-dependent information.

'A quantum nature', as we name, is every so-defined quantum universe-or-multiverse, being the entire one, a reduced one, or a renormalized one. As every universe is indeed a physically-realizable quantum many body system, we choose the initial system to be 'the ultimate quantum multiverse', defined as the set-theoretic union of all the in-principle-realizable universal quantum many body systems. Being so, it must be now manifest that *'all quantum natures'* are certainly re-identifable as all the 'physically-possible' quantum many body systems of the arbitrarily-chosen quantum objects and their arbitrarily-chosen quantum relations.

As a theory of physics, what this work is all about, is to construct from first principles the covariant total state-space and the covariant total Hamiltonian of the most-fundamental purely-information-theoretic quantum many body theory whose unitarily-evolving quantum states encode the most complete information of every such quantum nature. The HoloQuantum Network Theory which we present in this work does perfectly accomplish *this 'it-from-qubit' mission.*

II. HOLOQUANTUM NETWORK THEORY: THE THREEFOLD FOUNDATION AND THE QUANTUM STATICS

HoloQuantum Network Theory will be axiomatically both defined and formulated based upon nine principles. In making a theory, the choice of its principles is almost everything, so let us highlight a cardinal point about these nine principles. To the best of our understanding, the chosen principles are the ones which serve the theory as the unavoidable, and at the same time, as the very best foundational statements to fulfill its intentional mission as has been stated and explained in section one. Namely, *these nine principles are 'the must be' and 'the best be'.*

We now begin the work to answer the original question by stating the three principles which set the *'threefold foundation' of HoloQuantum Network Theory.*

Principle 1: HoloQuantum Network Theory, indicated by '$\mathcal{M}$', must be axiomatically defined and formulated to be the statically-and-dynamically covariant formulation of all the in-principle-realizable quantum many body systems whose defining quantum degrees of freedom are the arbitrarily-chosen quantum objects and their arbitrarily-chosen quantum relations, capturing their most-complete time-dependent states and observables. Stated equally, it must be the most complete theory of 'all quantum natures'. Mathematically, to accomplish the above mission, the theory $\mathcal{M}$ must be the most complete theory of HoloQuantum Hypergraphs, namely, all the hypergraphs whose defining structures, all being quantized, form as the quantum degrees of freedom a total distinctively-interacting quantum many body system with unitary quantum evolution.

Explanatory Note: We must highlight that, both the objects and their relations must be taken-in as the original notions and the quantum degrees of freedom of the presented theory. Because all relations, by their very conception, are to be defined between some objects, the theory, in order to be both the most fundamental and the most complete, does also need to have the quantum objects included in the totality of its microscopic degrees of freedom. But hierarchically viewed, relations can also be 'objectified' and so then develop the relations between relations. Being so, upon straightforward hierarchical extensions which keep the formulation of the theory invariant, the defining quantum degrees of freedom will additionally include all these *'hierarchical relational relations'*. Because these hierarchical extensions are all straightforward, they are left implicit in the present work.

Principle 2: To accomplish the mission specified in principle one, $\mathcal{M}$ must be an entirely-pregeometric quantum field theory, defined and formulated in $(0+1)$ spacetime dimensions. Namely, it must be 'a' theory of quantum mechanics.

Explanatory Note: HoloQuantum Network Theory, at the level of its fundamental formulation, lives in $(0+1)$ spacetime dimensions. However, its sub-theories, model extractions, solutions and phases realize all the in-principle-realizable many body systems of quantum objects and their quantum relations defined in arbitrary numbers of spatial dimensions.

Principle 3: $\mathcal{M}$ must be the most-fundamental 'it-from-qubit' theory of all quantum natures. Being so, HoloQuantum Hypergraphs must be formulated purely information-theoretically as the unitarily-evolving states of an unprecedented quantum many body system of the 'fundamental qubits', namely HoloQuantum Networks. By definition, each one of these fundamental qubits represents the states of the 'absence-or-presence' of a corresponding (sufficiently-refined) quantum vertex or a (sufficiently-refined) quantum hyperlink.

Explanatory Note: The qubits of $\mathcal{M}$ will be represented by means of the fermions. That is, the two defining basis-states of each qubit will be given by the eigenvalues of the number operator of a representing fermion, being zero or one. Of course, as an 'it-from-qubit' theory, the final computations of the true observables of the theory are independent of how one chooses to represent the qubits of the theory. Being so, these very same absence-or-presence qubits can be also represented by spins or by other alternatives. But, as the 'it-from-qubit' theory of all quantum natures, HoloQuantum Network Theory does embody its simplest first-rate formulation in terms of the *fermionically-represented qubits*. For simplicity, we shall name them *'fermion qubits'*.

We now merge these three principles into one unit:

The Threefold Foundation of $\mathcal{M}$:
$\mathcal{M}$ must be formulated as 'a' closed quantum mechanical system of the distinctively-interacting (fermion) qubits of the absences-or-presences of the arbitrarily-chosen quantum objects and their arbitrarily-chosen quantum relations. The global wavefunctions of all these fermion qubits must correspond in a one-to-one manner to the unitarily-evolving quantum superpositions of the arbitrarily-structured hypergraphs.

To formulate HoloQuantum Network Theory, let us choose an arbitrary $M \in \mathbb{N}$, and let $\mathcal{H}_M^{(\mathcal{M})}$ *be the defining total Hilbert space of all the HoloQuantum Hypergraphs whose time-dependent structures can contain up to M present vertices*. The defining degrees of freedom of $\mathcal{H}_M^{(\mathcal{M})}$ are a total number of $M^\star = M^\star(M)$ fermion qubits of the 'absences or presences', each one for a quantum vertex or for a quantum hyperlink. We take here a finite M, but are also free to take $M \to \infty$. Moreover, let us introduce an arbitrarily-chosen all-index '$\,I\,$' which *even-handedly* but uniquely labels all the $M^\star$ qubits as F_I. With every one qubit F_I come its creation and annihilation operators $(f_I, f_I^\dagger)$, forming the total fermionic algebra,

$$f_I^{\dagger 2} = f_J^2 = 0 \quad ; \quad \{f_J, f_I^\dagger\} = \delta_{JI} \quad , \quad \forall\, I \,,\, \forall\, J \qquad (1)$$

We name the *qubit* for a *vertex* a '*verton*', and the *qubit* for a *hyperlink* whose identification needs 'm' number of defining 'base vertices' a *'m-relaton'*, where $m \in \mathbb{N}^{\leq M}$. Because mathematically, a hypergraphs is defined free from any background space, $\mathcal{M}$ must be fundamentally defined in zero space dimensions. Being so, at the level of the fundamental formulation of the theory, vertons are labeled with one single index $i \in \mathbb{N}^{\leq M}$ to address their independent identities. We denote all the vertons of $\mathcal{M}$ by V_i, together with their fundamental operators $(v_i, v_i^\dagger)$. Accordingly, the m-relatons of $\mathcal{M}$ are addressed uniquely as $R_{i_{(m)}}$, and $(r_{i_{(m)}}, r^\dagger_{i_{(m)}})$, with $i_{(m)} \equiv \{i_1, ..., i_m\}$ being their *sets* of base-vertons. By any one-to-one map,

$$\{\text{all } i^{\in \mathbb{N}^{\leq M}}\} \cup \{\text{all } i_{(m \in \mathbb{N}^{\leq M})}\} \equiv \; \{I\}^{\text{Cardinality} \,=\, M^\star} \qquad (2)$$

Let us highlight an important point. In this section, given its mission, we must formulate only *'the basic class' of HoloQuantum Networks*. By definition, these are the HoloQuantum Networks for which the relatons are totally insensitive to the ordering of their base vertons, and moreover, the relatons and the vertons are weightless and flavorless. But, these basic-class-defining minimalizations will be all lifted up where *in section five*, on the basis of the results in sections two to four, *we will perfect both the mathematical and physical generality of HoloQuantum Network Theory by formulating the HoloQuantum Networks which are 'maximally flavored', 'maximally weighted', and also 'maximally oriented'.* Maximal orientedness means that the base-ordering degeneracy of the relatons are entirely lifted up, so that every relaton is defined by the unique ordering of all its base vertons.

Now, in the two labeling systems we have introduced, the fermion-qubit operators of $\mathcal{M}$ are so collected,

$$\{\ f_I^{(\dagger)}\ ;\ |\{I\}| = M^\star\ ;\ M^\star = 2^M - 1 + M\ \}\ \equiv \\ \equiv \{v_i^{(\dagger)}\ ;\ r_{i_{(1)}}^{(\dagger)}\ ,\ r_{i_{(2)}}^{(\dagger)}\ ,\ r_{i_{(3)}}^{(\dagger)}\ ,\cdots,\ r_{i_{(M)}}^{(\dagger)}\,|\ \text{all possible indices}\} \tag{3}$$

The simplest observables of the theory are the number operators of all the vertons and all the relatons,

$$n_I \equiv f_I^\dagger f_I \quad ; \quad n_I^2 = n_I \quad ; \quad [n_I, n_J] = 0 \quad , \quad \forall\, I\ ,\ \forall\, J \tag{4}$$

Multi-qubit states are defined as the common eigenstates of the n_I observables. Given the standard vacuum state,

$$|0\rangle \in \mathcal{H}_M^{(\mathcal{M})} \quad ; \quad f_I\,|0\rangle = 0 \quad , \quad \forall\, I \tag{5}$$

the '*m-qubit states*' are built as follows,

$$\prod_{I_s}^{1\leq s\leq m} f_{I_s}^\dagger\,|0\rangle\ \ ,\ \forall\,\{I_s\}_{|_{1\leq s\leq M}}^{\subset\{I\}^{\text{Cardinality } M^\star}} \quad ,\ \forall\, m \in \mathbb{N}^{\leq M^\star} \tag{6}$$

But, the defining state-space of the theory $\mathcal{M}$, namely $\mathcal{H}_M^{(\mathcal{M})}$, is much smaller than the space spanned by all the multi-qubit wavefunctions constructed in (6). This must be clear because, as one demands, *for a multi-qubit wavefunction in (6) to be a defining basis-state of $\mathcal{H}_M^{(\mathcal{M})}$, it must be identifiable with a mathematically-welldefined 'classical' hypergraph.* Restated, each multi-qubit state in (6) belongs to the defining basis of $\mathcal{H}_M^{(\mathcal{M})}$, if for every relaton $R_{i_{(m)}}$ being present in it, all of its base-vertons $\{V_{i_1}, \cdots, V_{i_m}\}$ are also present in it. This condition, which discerns the simultaneous basis of $\mathcal{H}_M^{(\mathcal{M})}$, mathematical represents the point that the presence of every relation without the presence of every single one of its identifying objects render its containing multi-qubit wavefunctions meaningless. This statement yields the set of *quantum constraints* whose 'statical form' comes in this section, but their 'dynamical form' will come in section three.

For every qubit F_I, let us introduce its uniquely-defined 'qubit-presence projection operator', being denoted by P_I. By definition, it is the unique operator by which any superposition of the states in (6) is projected down to the maximal part of it in which every participating state has the presence of the qubit F_I. By fermionic algebra,

$$P_I\ =\ n_I\ \ ;\ \ \bar{P}_I\ \equiv\ 1 - P_I\ =\ 1 - n_I \quad , \quad \forall\, I \tag{7}$$

Given this identification, here come the *'statical quantum constraints'* that discern which ones among the m-qubit wavefunctions in (6) form the basis of $\mathcal{H}_M^{(\mathcal{M})}$, $\mathcal{B}_M^{(\mathcal{M})}$,

$$\forall \text{ state } \left|\hat{\Psi}\right\rangle \text{ in } (6) \quad , \quad \left|\hat{\Psi}\right\rangle \in \mathcal{B}_M^{(\mathcal{M})} \quad ; \quad \text{if :} \\ Y_{i_{(m)}}\left|\hat{\Psi}\right\rangle\ \equiv\ n_{i_{(m)}}\,\Big(1\ -\prod_{\text{all } i_s\,\in\, i_{(m)}} n_{i_s}\Big)\left|\hat{\Psi}\right\rangle = 0 \quad , \quad \forall\, R_{i_{(m)}} \tag{8}$$

Constraints (8) are solved by replacing the $r_{i_{(m)}}^{(\dagger)}$s, one by one, with their 'vertonically-dressed' counterparts $\breve{r}_{i_{(m)}}^{(\dagger)}$s,

$$\breve{r}_{i_{(m)}}^{(\dagger)}\ \equiv\ r_{i_{(m)}}^{(\dagger)}\prod_{\text{all } i_s\,\in\, i_{(m)}} n_{i_s} \quad ; \quad \breve{n}_{i_{(m)}}\ =\ n_{i_{(m)}}\prod_{\text{all } i_s\,\in\, i_{(m)}} n_{i_s} \tag{9}$$

Being so defined, it must be clear that the operators (9) bring about only the *'hypergraph-state relatons'*, namely the 'correct relatons'. Keeping vertons intact, we now evenhandedly define the *'hypergraph-state-qubits'* $\breve{F}_I$ as,

$$\breve{f}_I\ \equiv\ f_I\prod_{\text{all the base } i_s \text{ for } I} n_{i_s}\ ;\ \breve{f}_I^\dagger\ \equiv\prod_{\text{all the base } i_s \text{ for } I} n_{i_s}\ \ f_I^\dagger \quad , \quad \forall\, I \tag{10}$$

Now, *the total Hilbert space of $\mathcal{M}$, namely $\mathcal{H}_M^{(\mathcal{M})}$, must be defined by its most-complete vertonic-and-relatonic tensor-product structure and its so-identified basis $\mathcal{B}_M^{(\mathcal{M})}$*,

$$\mathcal{H}_M^{(\mathcal{M})}\ =\ \mathcal{H}_{(\text{vertons})}^{(\mathcal{M})}\ \otimes_{1\leq m\leq M}\ \mathcal{H}_{(m-\text{relatons})}^{(\mathcal{M})} \\ \mathcal{B}_M^{(\mathcal{M})}\ =\ \Big\{\ |I_1\cdots I_m\rangle\ \ ,\ \forall\,\{I_s\}_{|_{1\leq s\leq m}}^{\subset\{I\}^{\text{Cardinality } M^\star}}\ ,\ \forall\, m\in\mathbb{N}^{\leq M^\star}\Big\} \\ (\text{where :}) \quad |I_1\cdots I_m\rangle\ \equiv\ \left|\vec{I}_{\|m\|}\right\rangle\ \equiv\ \prod_{I_s}^{1\leq s\leq m}\breve{f}_{I_s}^\dagger\ \ |0\rangle \tag{11}$$

By the second and the third lines of (11), the basis $\mathcal{B}_M^{(\mathcal{M})}$ is defined to be the set of the common-eigenstates of all the number operators n_Is in which a possible collection of the hypergraph-state-qubits $\{\breve{F}_{I_1}\cdots\breve{F}_{I_m}\}$ has taken presence, while all the other hypergraph-state-qubits $\{\breve{F}_I\}^{(\text{Cardinality } M^\star)} - \{\breve{F}_{I_1}\cdots\breve{F}_{I_m}\}$ are absent. Being so, every $\left|\vec{I}_{\|m\|}\right\rangle^{\in\,\mathcal{B}_M^{(\mathcal{M})}}$ corresponds to a unique 'classical' hypergraph, and so identifies the set of all $\left|\vec{I'}_{\|m\|}\right\rangle^{\in\,B_M^{(\mathcal{M})}}$ which as classical hypergraphs are isomorphic to it. We will elaborate on this classification in principle five.

Result (11), *concludes the total quantum statics of* $\mathcal{M}$. But, let us highlight two points. *First*, HoloQuantum Network Theory is, and must be, a 'fully-internal theory', with no 'external' qubits in it whatsoever. All its qubits correspond information-theoretically to the defining structure of the HoloQuantum Hypergraphs. Vertons are qubits for objects, the vertices, and relatons are qubits for relations, the hyperlinks. Clearly, there can be no other qubits in an 'it-from-qubit' theory of all quantum natures. *Second,* HoloQuantum Network Theory is the theory 'of' the mathematically-quantized hypergraphs, not a quantum theory 'on' hypergraphs. All possible backgrounds and excitons appear only as sub-theories and models, solutions and phases of $\mathcal{M}$.

As of now, our task is developing axiomatically all the many-body interactions, and the total Hamiltonian of HoloQuantum Networks, which determine both their complete unitary dynamics and phase diagram.

III. HOLOQUANTUM NETWORK THEORY: THE FUNDAMENTAL RULE AND CASCADE OPERATORS

Let us now begin to work out *the dynamical laws* of $\mathcal{M}$. In this section, based on the next five principles, principles four to eight, we formulate *'the fundamental rule' of* $\mathcal{M}$, namely, its core many-body interactions.

HoloQuantum Network Theory, at the fundamental level, must be the theory of the *closed* quantum many body system of all the vertons and all the relatons. This 'closed-system criterion', which is already imposed by principle one, comes on two regards. *Firstly*, for being the quantum many body theory of all quantum natures, the above criterion must be met necessarily, because every quantum universe-or-multiverse must be a closed many body system. *Secondly*, this feature is a prerequisite to the theory to formulate all the forms of HoloQuantum Networks, fulfilling its mathematical completeness. Because, by having 'closed HoloQuantum Networks' primarily, the 'open HoloQuantum Networks' will be formulated by a general partitioning of the total system of vertons-and-relatons, and deriving the master equation out of the total unitary evolution in $\mathcal{M}$. Being a closed quantum system of all $\breve{F}_I$ s, the total evolution of the HoloQuantum Networks must be *this unitary one*,

$$U_M^{(\mathcal{M})}(t) \equiv e^{itH_M^{(\mathcal{M})}} \; ; \; H_M^{(\mathcal{M})} \equiv \sum_{\kappa}^{\text{all}} \lambda_\kappa \, \mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})}[\{\breve{f}_J\} \; ; \; \{\breve{f}_I^\dagger\}] \tag{12}$$

By the right-hand side of (12), the *total Hamiltonian* of $\mathcal{M}$, $H_M^{(\mathcal{M})}$, must be a sum over *all the 'Hamiltonian Operators'* $\mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})}$ which, as composite operators made from the f_Js and the $f_I^\dagger$s, realize all the many-body interactions between the vertons-and-relatons.

We will *uniquely* determine (12), *by realizing the nine principles*. Going forward, we set the next four principles.

Principle 4: The hypergraph-state well-definedness of HoloQuantum Networks must be preserved during their entire unitary evolutions whose generating Hamiltonian, in its maximally-general form, is expressed as (12).

Explanatory Note: Remember that HoloQuantum Networks are the unitarily-evolving superpositions of the $\mathcal{B}_M^{(\mathcal{M})}$-states given in (11). *Being central to* $\mathcal{M}$, principle four demands that, when $U_M^{(\mathcal{M})}(t)$, whose most general form is given in (12), acts on each basis-state in (11) chosen as an initial state, the 'vertonically-unsupported relatons' should not take presence in the wavefunctions. The resulted *dynamical quantum constraints* will be solved by a whole family of *'Cascade Operators'*.

Principle 5: In HoloQuantum Network Theory, there must be a complete set, $\mathcal{S}_M^{(\mathcal{M})}$, of the operators of the 'second-quantized hypergraphical isomorphisms', all being made from some of the exact Hamiltonian operators $\mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})}$ in (12). These operators complete the 'isomorphism-base' purely-vertonic conversions by all their induced relatonic adjustiments. By the actions of these operators on HoloQuantum Networks, they are transformed into their hypergraphically-isomorphic states, in such a way that every contributing basis-state (11), as a 'classical' hypergraph, either remains invariant automorphically, or is transformed isomorphically.

Explanatory Note: $\mathcal{M}$ calls for principle five by being mathematically *'the quantum theory of hypergraphs'*. We so define how the above quantum isomorphisms, which form $\mathcal{S}_M^{(\mathcal{M})}$, must be realized in the second-quantized HoloQuantum Networks. First, one defines an abstract discrete space $\mathcal{V}_M^{(\mathcal{M})}$, the 'vertonic field space', for which the indices of all the M vertonic degrees of freedom of $\mathcal{H}_M^{(\mathcal{M})}$ form a 'coordinate system'. Now, every element of $\mathcal{S}_M^{(\mathcal{M})}$ is a permutational coordinate transformation on $\mathcal{V}_M^{(\mathcal{M})}$, $\{V_i\} \to \{V_{\mathcal{P}(i)}\}$, being accompanied by all the 'induced' adjustments on the indices of relatons, $\{R_{i_{(m)}}\} \to \{R_{\mathcal{P}(i)_{(m)}}\}$. Realized as field operations in the second-quantized framework, such transformations are all realized by the specific products of the so-constructed *'order-one isomorphism operators'*, Γ_j^i,

$$\begin{aligned} \Gamma_j^i \; &\equiv \; \mathbb{1} \, \bar{P}_j \; + \; [\; \mathbb{1} \, P_i \; + \; (v_j \mathcal{G}_j^i v_i^\dagger) \, \bar{P}_i \;] \, P_j \; = \\ &= \; 1 + n_j(1-n_i) \, (v_j \mathcal{G}_j^i v_i^\dagger - 1) \quad , \, \forall \, (j,i) \end{aligned} \tag{13}$$

which act on all the HoloQuantum-Networks $|\Psi\rangle^{\in \mathcal{H}_M^{(\mathcal{M})}}$. In (13), the permutation '$j \to i$' is relazied by the purely-vertonic operator $v_j v_i^\dagger$ and is simultaneously completed as an isomorphism by the purely-relatonic operator $\mathcal{G}_j^i$, fulfilling the relaton adjustments as needed by the vertonic inductions.

Principle 6: The global $U(1)$ transformations of the hypergraph-state-qubit operators $f_I^{(\dagger)}$ which generate global phases on the multi-qubit states of $\mathcal{B}_M^{(\mathcal{M})}$ in (11) should be all redundant observationally. Being so, these transformations form a quantum-mechanical global $U(1)$ fundamental symmetry of $\mathcal{M}$ under which all the observables, states, many-body interactions, and the unitary evolutions of HoloQuantum Networks must be invariant.

Explanatory Note: HoloQuantum Network Theory is an inherently background-less quantum field theory. As such, considering the theory at its fundamental level, there can be no spatial-background potential or non-trivial spatial-background topology by which the global phases of the multi-qubit wavefunctions can become observables. On the other hand, by the identification of $\mathcal{B}_M^{(\mathcal{M})}$ in (11), these redundant phases of the basis-states are all generated by the *global* $U(1)$ transformations on the fundamental operators of the hyper-state-qubits $f_I^{(\dagger)}$s, which so must be the global $U(1)$ symmetry transofrmations of $\mathcal{M}$.

Principle 7: HoloQuantum Network Theory, at the level of its fundamental formulation, must treat all the m-relatons, independent of their defining hyperlink degrees m, in an exactly-equal manner. Moreover, even when incorporating the vertons and the relatons all together, still all the hypergraph-state-qubits $\breve{F}_I$ must be treated with the 'maximum-possible level of equality' everywhere in the whole theory. Namely, this 'all-qubits equal-treatment' is explicitly and minimally broken by only the statical and dynamical quantum constraints for the hypergraph-state dependencies of the relatons to their defining vertons. This will be a minimally-broken symmetry which nevertheless cardinally serves the maximum-hypergraphical construction of $\mathcal{M}$.

Explanatory Note: By realizing this minimally-broken symmetry, *the maximum level of the 'hypergraphic-ness'* is, staticaly and dynamically, realized in $\mathcal{M}$, albeit at the level of its fundamental formulation. Being a *cartdinal feature* of HoloQuantum Network Theory, this characteristics is 'a must' for its fulfilment as the most complete covariant theory of all quantum natures. This becomes manifested in the following three statements. *First*, two-relatons are not the only relatons or the more special relatons of $\mathcal{H}_M^{(\mathcal{M})}$, in distinction with 'graphical theories'. *Second*, all the m-relatons participate in the many-body interactions of the theory, and so in its total Hamiltonian, with no exclusion, and in an absolutely-equal manner. *Third*, this principle implies that, by some many-body interactions of the theory the vertons and the m-relatons must be converted to one another, in all possible conversional channels, as long as the complete hypergraphical well-definedness of the HoloQuantum Networks are preserved.

Principle 8: $\mathcal{M}$, in order to be the most-fundamental and the most-complete quantum many body theory of all quantum natures, must be both 'fundamentally random' and 'maximally random'. Namely, at the level of the fundamental formulation of $\mathcal{M}$, absolutely all the many-body interactions must be defined up to the maximally-random ensembles of their defining couplings.

Explanatory Note: $\mathcal{M}$, by its original intention of being the most fundamental theory of all quantum natures, must be constructed in *'the maximal resonance'* with the statistical randomness which is at the centre of the *principle of 'Law Without Law' by Wheeler* [1]. This also resembles the randomness in the matrix-and-tensor models [2]. HoloQuantum Network Theory does realize this fundamental randomness 'maximally'. That is, $H_M^{(\mathcal{M})}$ recieves absolutely all the many-body-interaction operators $\mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})}$ by the couplings λ_κ whose statistical distributions are 'maximally random'. We will choose to take all the couplings to be (the real-or-complex valued) continuous parameters, although they may be also taken to be parameters with discrete spectra. By this choice, the maximally-free Gaussian distributions for all the λ_k serve the formulation of $\mathcal{M}$ as the most natural ones. Principle eight supports the 'embedding completeness' of the theory additionally, because then all the deterministic theories are also embeddable in it, by suitably fixing the two characteristic parameters of the corresponding maximally-free Gaussian distributions.

Already before the step-by-step construction of $H_M^{(\mathcal{M})}(t)$, we state *The Fundamental Rule of* $\mathcal{M}$ which is obtained later in this section by *merging principles one to eight*.

The Fundamental Rule of $\mathcal{M}$:
By the first-order many-body interactions of $\mathcal{M}$, namely by its core microscopic operations, every arbitrary pair of the hypergragh-state-qubits $(\breve{F}_J, \breve{F}_I)$, either one being any verton or any m-relaton, can become converted to one another, $\breve{F}_J \leftrightarrows \breve{F}_I$. The corresponding strengths of these core operations are set by the maximally-free Gaussian-random couplings $(\bar{\lambda}_J^I, \lambda_J^I)$. If the conversion is purely relatonic, $\breve{r}_{j_{(m_j)}} \to \breve{r}_{i_{(m_i)}}$, no accompanying conversions are required. But, if the conversion pair contains a verton, $v_j \leftrightarrows \breve{f}_I$, a cascade of purely-relatonic conversions will be jointly triggered. These companion conversions, which secure the dynamical hypergraphical well-definedness of HoloQuantum Networks, are realized by the 'First-Order Cascade Operators' $(\mathcal{C}_j^{I\dagger}, \mathcal{C}_j^I)$.

By realizing the above eight (out of nine) principles, we now commence to obtain the first-order many-body interactions of HoloQuantum Network Theory, namely the above fundamental rule. But also, *the correct forms* of the first-order cascade operators and the first-order Hamiltonian of $\mathcal{M}$ are determined. This shows the road map to construct $U_M^{(\mathcal{M})}(t)$ by realizing principle nine.

Let us begin this mission with *realizing principle four*. We must take the initial HoloQuantum Network to be an arbitrary superposition of the basis-states in (11). This initial wavefunction does clearly satisfy all the constraints (11), because of being a superposition of the well-defined hypergraphs. Principle four demands that the resulted HoloQuantum-Network wavefunction at arbitrary time t, as obtained by the action of the $U_M^{(\mathcal{M})}(t)$ given in (12), must be also hypergraphically weldefined, that is, still a superposition of the very same basis-states in (11). On one hand, this simply means that the evolution-generating Hamiltonian of the general form in (12), $H_M^{(\mathcal{M})}[\{\breve{f}_J\}_{|_{\forall J}};\{\breve{f}_I^\dagger\}_{|_{\forall I}}]$, must serve $\mathcal{M}$ as an operator defined consistently on the Hilbest space (11), so that its actions do stay inside it. That is, its 'basis-dependent expression' must be of the form,

$$H_M^{(\mathcal{M})}[\{\breve{f}_J\}_{|_{\forall J}};\{\breve{f}_I^\dagger\}_{|_{\forall I}}] = \sum_{\vec{J}_{\|s\|},\vec{I}_{\|r\|}} h(\vec{J}_{\|s\|}|\vec{I}_{\|r\|}) \left|\vec{I}_{\|r\|}\right\rangle \left\langle \vec{J}_{\|s\|}\right| \tag{14}$$

On the other hand, we need to obtain the form of $H_M^{(\mathcal{M})}$ which, *first*, manifestly satisfies all the nine principles, and *second*, is directly, solely, and basis-independently formulated as an explicit composite operator made of all the 'alphabet' operators $\{\breve{f}_J\}_{|_{\forall J}} \cup \{\breve{f}_I^\dagger\}_{|_{\forall I}}$, as meant by the expression in the right hand side of (12). Aiming at this alternative form, let us now directly work out all that is explicitly phrased in principle four. Being stated there, all the HoloQauntum-Network wavefunctions which are resulted from acting the time-t evolution operator (12) on the initial states spanned by the basis (11), should also satisfy the constraints (8),

$$\begin{aligned} &\forall\ |\Psi\rangle \quad \text{satisfying} \quad Y_{i_{(m)}}\,|\Psi\rangle = 0 \ \ ,\ \forall\, R_{i_{(m)}} \\ &\text{also must have}:\ Y_{i_{(m)}}\,\left(U_M^{(\mathcal{M})}(t)\,|\Psi\rangle\right) = 0 \ \ ,\ \ \forall\, R_{i_{(m)}}\ ,\ \forall\, t \end{aligned} \tag{15}$$

Because $U_M^{(\mathcal{M})}(t)$ is generated by $H_M^{(\mathcal{M})}$, the conditions (15) can be equally restated as the following conditions,

$$[H_M^{(\mathcal{M})}, Y_{i_{(m)}}] \ \propto\ \text{function of the constraints } Y_{j_{(s)}}\ ,\ \ \forall\, R_{i_{(m)}} \tag{16}$$

Every operator $\mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})} \equiv \mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})}[\{\breve{f}_J\};\{\breve{f}_I^\dagger\}]$ in (12) is an individual term of the total Hamiltonian. Being so, because of (16), it must also satisfy the constraints,

$$[\mathcal{O}^{(H_M^{(\mathcal{M})})}, Y_{i_{(m)}}] \propto\ \text{function of the constraints } Y_{j_{(s)}}\ ,\forall\, R_{i_{(m)}} \tag{17}$$

We call the set of constraints (16) or (17) the *'dynamical quantum constraints' of* $\mathcal{M}$. Now, to realize principle four for HoloQuantum Networks, we obtain the solutions to the above constraints (17). Besides the simplest but important solutions which satisfy (17) trivially, we obtain here *all the cascade-operator-solutions of the one-to-one conversions*. Their generalizations will be given in next section by means of the 'descendant cascade' operators.

Clearly enough, the most straightforward family of the general solutions to (17) are given by *the purely-relatonic many-body operators*, namely by the following operators,

$$\mathcal{O}^{(\text{purely relatonic})}_{\{j_{a'(s_{a'})}\},\{i_{a(r_a)}\}} \equiv \Big(\prod_{1\le a'\le m'} \breve{r}_{j_{a'(s_{a'})}}\Big)\Big(\prod_{1\le a\le m} \breve{r}^\dagger_{i_{a(r_a)}}\Big) \tag{18}$$

The complementary class of solutions to (17) will be given by *the many-body operators which contain in them an arbitrary number of the verton operators*. Such a vertonically-involved many-body operator generically does not satisfy all the dynamical constraints in (17). This must be clear because, if a vertonic annihilation operator v_j acts on a basis-state in (11), generically it turns it into a hypergraphically-incorrect multi-qubit wavefunction, as all the V_j-based relatons which were present in the at-the-time state of the system, would be now leftover with incomplete vertonic support. Hence, every such v_j-action must be accompanied by a cascade operator whose defining action protect the resulted state against the vertonically-unsupported relatons. Here we obtain the *first-order cascade operators*.

First, let us consider the operator $\mathcal{O}^{(H_M^{(\mathcal{M})})}$ by which a verton V_j is converted to a verton V_i. Because the simplest option $v_j v_i^\dagger$ can not be a Hamiltonian operator, we must enlarge it by a yet-unknown *cascade operator* $\mathcal{C}_j^i$, and demand that the operator $v_j\mathcal{C}_j^i v_i^\dagger$ satisfies (17). We now determine the *the correct form of the cascade operator* $\mathcal{C}_j^i$. To fulfill its intented mission, $\mathcal{C}_j^i$ must be divisible into the following product form,

$$\mathcal{C}_j^i \ =\ \prod_{m=1}^{M}\ \prod_{y_{(m)}\,\equiv\,\{y_1\cdots y_m\}}^{\text{all choices}} \mathcal{C}_{y_{(m)}}^{j\to i} \tag{19}$$

Every contributing $y_{(m)}$ in the right-hand side of the above product (19), identifies an arbitrary choice of m vertons $\{V_{y_1},\cdots,V_{y_m}\}$ out of the $M-2$ available vertons setting aside $\{V_i,V_j\}$. In (19) all such choices of $y_{(m)}$s are incorporated impartially. Moreover, all the internal operators $\mathcal{C}_{y_{(m)}}^{j\to i}$ must be expandable in this form,

$$\begin{aligned} \mathcal{C}_{y_{(m)}}^{j\to i} \ &=\ A_\perp^\perp\ \bar{\breve{P}}_{iy_{(m)}}\bar{\breve{P}}_{y_{(m)}j} \ +\ A_\perp^\|\ \bar{\breve{P}}_{iy_{(m)}}\breve{P}_{y_{(m)}j} \\ &+\ \ A_\|^\perp\ \breve{P}_{iy_{(m)}}\bar{\breve{P}}_{y_{(m)}j} \ +\ A_\|^\|\ \breve{P}_{iy_{(m)}}\breve{P}_{y_{(m)}j} \end{aligned} \tag{20}$$

In (20), the indices '$y_{(m)}j$' and '$iy_{(m)}$' identify the two $(m+1)$ relatons $\breve{R}_{y_{(m)}j}$ and $\breve{R}_{iy_{(m)}}$ which relate the m vertons $\{V_{y_1},\cdot\cdot\cdot,V_{y_m}\}$ to V_j and V_i, respectively. Also, $(\breve{P}_{y_{(m)}j},\bar{\breve{P}}_{y_{(m)}j})$ and $(\breve{P}_{iy_{(m)}},\bar{\breve{P}}_{iy_{(m)}})$ are the doublets of the projection operators which respectively correspond to the presences and the absences of $\breve{R}_{y_{(m)}j}$ and $\breve{R}_{iy_{(m)}}$. Furthermore, there are two sets of operator identities that all the $\mathcal{C}_j^i$s, and so as induced by them, each one of the $\mathcal{C}_{y_{(m)}}^{j\to i}$ s, must satisfy. These identities are as follows,

$$(\mathcal{C}_j^i)^\dagger = \mathcal{C}_i^j \ \Rightarrow\ \left(\mathcal{C}_{y_{(m)}}^{j\to i}\right)^\dagger = \mathcal{C}_{y_{(m)}}^{i\to j}\ ;\quad \mathcal{C}_j^j = 1 \ \Rightarrow\ \mathcal{C}_{y_{(m)}}^{j\to j} = 1 \tag{21}$$

By the left-side of (21), the two inversely-conversional operators $\mathcal{O}_j^{(H_M^{(\mathcal{M})})i} \equiv v_j \mathcal{C}_j^i v_i^\dagger$ and $\mathcal{O}_i^{(H_M^{(\mathcal{M})})j} \equiv v_i \mathcal{C}_i^j v_j^\dagger$ add up to a Hermitian unit of $H_M^{(\mathcal{M})}$. Right-side condition in (21), on the other hand, must be required for the following 'interaction-consistency criterion' to be realized. The number operator $n_j = v_j^\dagger v_j$, which surely is a Hamiltonian operator by satisfying (17), can be regarded as the conversion of V_j into the same V_j, and so can be likewise recast as $n_j = 1 - v_j \mathcal{C}_j^j v_j^\dagger$, yielding $\mathcal{C}_j^j = 1$.

By applying the two sets of the operator identities in (21) to the general expansion in (20), one obtains,

$$\begin{aligned} \mathcal{C}_{y_{(m)}}^{j\to i} \;&=\; \big(\bar{\breve{P}}_{iy_{(m)}} \bar{\breve{P}}_{y_{(m)}j} + \breve{P}_{iy_{(m)}} \breve{P}_{y_{(m)}j}\big) \\ &+\; A_\perp^\parallel \; \bar{\breve{P}}_{iy_{(m)}} \breve{P}_{y_{(m)}j} + \; (A_\perp^\parallel)^\dagger \; \breve{P}_{iy_{(m)}} \bar{\breve{P}}_{y_{(m)}j} \end{aligned} \tag{22}$$

Now, for the $V_j \leftrightarrows V_i$ conversions to realize *principle four*, one demands that the enlarged operators $(v_j \mathcal{C}_j^i v_i^\dagger, v_i \mathcal{C}_i^j v_j^\dagger)$ satisfy the dynamical constraints (17). By satisfying this requirement together with realizing *principle five* in the corresponding form which realizes (13), and finally upon utilizing $(\breve{P}_I, \bar{\breve{P}}_I) = (\breve{n}_I, 1 - \breve{n}_I)$, we conclude as follows *the purely-vertonic first-order cascade operators* $\mathcal{C}_j^i$,

$$\begin{aligned} \mathcal{C}_j^i \;&=\; \prod_{m=1}^{M} \; \prod_{y_{(m)}}^{\text{all choices}} \; \mathcal{C}_{y_{(m)}}^{j\to i} \\ \mathcal{C}_{y_{(m)}}^{j\to i} \;&=\; \big(1 - \breve{n}_{iy_{(m)}} - \breve{n}_{y_{(m)}j} + 2\, \breve{n}_{iy_{(m)}} \breve{n}_{y_{(m)}j}\big) \\ &+\; (1 - \delta_{ij})\; \big(\breve{r}_{y_{(m)}j}^\dagger \; \breve{r}_{iy_{(m)}} + \breve{r}_{iy_{(m)}}^\dagger \; \breve{r}_{y_{(m)}j}\big) \end{aligned} \tag{23}$$

By fermionic algebra and because C_i^j appears in $H_M^{(\mathcal{M})}$ always inside its enlarged operator $v_j C_j^i v_i^\dagger$, one sees that, *the cascade operators obtained as* (23), *can be equivalently simplified in the following form*,

$$\mathcal{C}_j^i \;=\; \prod_{m=1}^{M} \; \prod_{y_{(m)}}^{\text{all choices}} \; \big[\; \big(1 \;-\; \breve{n}_{y_{(m)}j}\big) \;+\; \breve{r}_{iy_{(m)}}^\dagger \; \breve{r}_{y_{(m)}j} \;\big] \tag{24}$$

Next, we formulate the *conversion between a verton and a relaton*. Likewise above, we so introduce the Hamiltonian operator $v_j \mathcal{C}_j^I \breve{f}_I^\dagger$ to realize the conversion of a verton V_j to *any* hypergraph-state-qubit $\breve{F}_I$. By realizating *principle seven*, namely maximal hypergraphness, the operator $\mathcal{C}_j^I$ must be so identified,

$$\mathcal{C}_j^I \equiv \; \mathcal{C}_j^k \; [\text{ index } k \text{ being replaced with the index } I \;] \;, \; \forall \; I \tag{25}$$

with $V_{k\neq j}$ being whatever verton. Therefore, we obtain,

$$\mathcal{C}_j^I \;=\; \prod_{m=1}^{M} \; \prod_{y_{(m)}}^{\text{all choices}} \; \big[\; \big(1 - \breve{n}_{y_{(m)}j}\big) + \; \breve{r}_{Iy_{(m)}}^\dagger \; \breve{r}_{y_{(m)}j} \;\big] \tag{26}$$

with $R_{Iy_{(m)}}$ being the unique relaton whose indices are the union of $y_{(m)}$ with all the vertonic indices of F_I.

Indeed, one can now directly see that upon the insertion of the *'all-species cascade operator'* (26), $v_j \mathcal{C}_j^I \breve{f}_I^\dagger$ becomes a hypergraphically-acceptable conversion operator, as its satisfies the constraints (17). We highlight that, the inverse conversion $F_I \to V_j$ is conducted by its Hermitian-conjugate cascade operator, that is, by $\mathcal{C}_I^j = (\mathcal{C}_j^I)^\dagger$. *The result* (26) *completes the axiomatic construction of the first-order cascade operators of* $\mathcal{M}$.

Let us now confirm that indeed principle five has been correctly realized in $\mathcal{M}$. By this principle, the complete set of the second-quantized hypergraphical isomorphisms, $S_M^{(\mathcal{M})}$, must be realized in the theory as specified both in the statement of principle five and in its explanatory note. *First*, the quantum-hypergraphical isomorphisms must be identified with some composite operators which are constructed from a proper subset of the multi-qubit-conversion operators of $\mathcal{M}$, that is, *from a specific class of the Hamiltonian operators* $\mathcal{O}_\kappa^{H_M^{(\mathcal{M})}}$ *as introduced in* (12). *Second*, every one of these operators must complete a corresponding many-body conversion in $\mathcal{M}$ which is *purely vertonic*. Being mathematically represented, the purely-vertonic conversions do realize all the coordinate permutations on $\mathcal{V}_M^{(\mathcal{M})}$, in a one-to-one manner. Being so, to form the generators Γ_j^is (13), they must be completed by the induced relatonic adjustments. Looking into $\mathcal{C}_j^i$ in (24), one indeed confirms that,

$$\begin{aligned} \mathcal{G}_j^i \;&=\; \mathcal{C}_j^i \qquad , \qquad \forall \;\; (j, i) \\ \Gamma_j^i \;&=\; 1 + n_j(1 - n_i)(v_j \mathcal{C}_j^i v_i^\dagger - 1) \end{aligned} \tag{27}$$

Being generated by the operators of the first-order quantum-hypergraphical isomorphisms given in (27), the 'order-$m^{\in \mathbb{N}^{\leq M}}$' elements of $\mathcal{S}_M^{(\mathcal{M})}$ are so constructed,

$$\begin{aligned} \Gamma_{j_1\cdots j_m}^{i_1\cdots i_m} \;&\equiv\; \Gamma_{j_1}^{i_1} \;\cdots\; \Gamma_{j_m}^{i_m} \;= \\ &=\; \prod_{s}^{1\leq s\leq m} \; [\; 1 + n_{j_s}(1 - n_{i_s})(v_{j_s} \mathcal{C}_{j_s}^{i_s} v_{i_s}^\dagger - 1) \;] \end{aligned}$$

$$\Gamma_{j_1\cdots j_m}^{i_1\cdots i_m} \, |\Psi\rangle^{\in \, \mathcal{H}_M^{(\mathcal{M})}} \;=\; \big|\Psi'\big\rangle^{\in \, \mathcal{H}_M^{(\mathcal{M})}} \;\cong\; |\Psi\rangle^{\in \, \mathcal{H}_M^{(\mathcal{M})}} \quad , \; \forall \; |\Psi\rangle^{\in \, \mathcal{H}_M^{(\mathcal{M})}} \tag{28}$$

But, we highlight a very important characteristic of $\mathcal{M}$. Primarily, the cascade operators $\mathcal{C}_J^I$ are introduced in the theory to secure the hypergraphical well-definedness of the HoloQuantum Networks during their evolutions. Being so, setting aside all the purely-vertonic ones, the 'mixed-species cascade operators' $(\mathcal{C}_j^{i_{(m)}}, \mathcal{C}_{i_{(m)}}^j)$, which are obtained in (26), do not realize any such isometries in $\mathcal{M}$. That is, their purely-relatonic cascades which complete the verton-relaton conversions $V_j \leftrightarrows R_{i_{(m)}}$, do transmute the HoloQuantum Networks non-isomorphically. Let us restate the crucial point. *The conversions triggered by the mixed-species cascade operators form (only a proper) subset of the immensely large set of the 'transmutational interactions' in* $\mathcal{M}$.

Next, we advance to the realization of principle six. This principle defines the quantum-mechanical symmetry of HoloQuantum Network Theory. Re-highlighting a point of distinction, we must remind that principle five is sourced by the mathematical character of the theory. On the other hand, principle six originates from the physical character of $\mathcal{M}$, being a theory of quantum mechanics. This principle is the statement that the global phases of the basis-states of $\mathcal{H}_M^{(\mathcal{M})}$ must be unobservables. Because all the multi-qubit states of $\mathcal{M}$ are created as (5,10,11) identify, the following complete set of the global $U(1)$ transformations on the hypergraph-state-qubits F_Is,

$$\breve{f}_I \to \exp(-i\phi)\ \breve{f}_I \quad ; \quad \breve{f}_I^\dagger \to \exp(+i\phi)\ \breve{f}_I^\dagger \quad , \ \forall\ \phi \in \mathbb{R}\ ,\ \forall\ I \tag{29}$$

which develop those global phases, must be symmetry transformations. All the observables of the theory must be the singlets of the global $U(1)$ transformations (29) which assign charges $(-1,+1)$ to the operator-doublets $(\breve{f}_I, \breve{f}_I^\dagger)$. That is, every Hamiltonian operator $\mathcal{O}^{(H_M^{(\mathcal{M})})}$ must be composed from the products of equal numbers of the $\breve{f}_J$s and the $\breve{f}_I^\dagger$s, to be $U(1)$-chargeless. Imposing this condition on the solutions of the quantum constraints (17) obtained so far, we conclude that all the following operators are the microscopic interactions of $\mathcal{M}$,

$$\begin{aligned}&\{\ \mathcal{O}^{(H_M^{(\mathcal{M})})}\ \}\ =\ \{\ \mathcal{O}^{(H_M^{(\mathcal{M})})}[\ \{\breve{f}_J\ ;\ \{\breve{f}_I^\dagger\}\ ]\ \}\ \supset\\ &\supset\ \{\ \big(\breve{r}_{j_{1_{(s_1)}}} \dots \breve{r}_{j_{m_{(s_m)}}}\big)\big(\breve{r}^\dagger_{i_{m_{(r_m)}}} \dots \breve{r}^\dagger_{i_{1_{(r_1)}}}\big)\ ;\ v_j \mathcal{C}_j^I \breve{f}_I^\dagger\ ;\quad h.c\ \}\end{aligned} \tag{30}$$

Result (30) does summarize the fundamental rule of $\mathcal{M}$. By this result, the 'first-order' Hamiltonian operators of HoloQuantumNetwork Theory are concluded as follows,

$$\{\ \mathcal{O}^{(H_M^{(\mathcal{M})})}_{(\text{first order})}\ \}\ =\ \{\ \mathcal{O}^{(H)I}_{(1)J}\ \equiv\ \breve{f}_J\, \mathcal{C}_J^I\, \breve{f}_I^\dagger \quad ,\quad \forall\ J\ ,\ \forall\ I\ \} \tag{31}$$

Let us summarize as follows the cascade operators $\mathcal{C}_J^I$ of the first-order Hamiltonian operators (31),

$$\begin{aligned}&\mathcal{C}_J^I \in \{\ \mathcal{C}_j^I\ ,\ \mathcal{C}_{\breve{r}_{j_{(n)}}}^{\breve{r}_{i_{(m)}}}\ \}\\ &\mathcal{C}_{\breve{r}_{j_{(n)}}}^{\breve{r}_{i_{(m)}}} = 1 \quad ;\quad \mathcal{C}_j^j = 1\\ &\mathcal{C}_j^{I\neq j} = \prod_{m=1}^{M}\ \prod_{w_{(m)}}^{\text{all choices}}\ \big(1 - n_{w_{(m)}j} + \breve{r}^\dagger_{Iw_{(m)}} \breve{r}_{w_{(m)}j}\big)\end{aligned} \tag{32}$$

By the findings (31,32), we so obtain the *first-order Hamiltonian of HoloQuantum Network Theory*,

$$H_M^{(\mathcal{M})(1-1)} = \sum_{I,J} \lambda_J^I\, \mathcal{O}^{(H)I}_{(1)J} \ =\ \sum_{I,J} \lambda_J^I\ \ \breve{f}_J\, \mathcal{C}_J^I\, \breve{f}_I^\dagger \tag{33}$$

where for the evolution to be unitary, we demand,

$$\lambda_I^J = \bar{\lambda}_J^I \tag{34}$$

Now, we must realize principle eight which plays a very central role in fulfilling the intention of HoloQuantum Network Theory to be the most fundamental quantum many body theory of all quantum natures. By this intrinsically-Wheelerian principle of $\mathcal{M}$, at the level of its fundamental formulation, the couplings of all the $\mathcal{O}_\kappa^{(H_M^{(\mathcal{M})})}$ Hamiltonian operators must be implemented to be maximally random. Upon applying this statement to the first-order Hamiltonian of HoloQuantum Network Theory, all of the independent couplings λ_J^I in the result (33) must be uncorrelated random parameters defined with their *maximally-free Gaussian distributions*,

$$\mathcal{P}(\lambda_J^I \mid \mathring{\lambda}_J^I\ ,\hat{\lambda}_J^I) \sim \exp\big(-\frac{(\lambda_J^I - \mathring{\lambda}_J^I)^2}{\hat{\lambda}_J^I}\big) \tag{35}$$

The characteristic parameters $(\mathring{\lambda}_J^I, \hat{\lambda}_J^I)$ in (35) by which the statistical mean values and mean squared values of λ_J^Is are respectively determined, take *arbitrary values*.

Let us now classify microscopically the interactions of the first-order Hamiltonian (33). As the summation which defines $H_M^{(\mathcal{M})}$ in (33) is all-inclusive, all the index degeneracies are contained in it. Being so, for every hypergraph-state-qubit $\breve{F}_I$, there comes an independent maximally-free Gaussian-random chemical-potential term $\mu_I \equiv \lambda_I^I$. That is, by discerning the qubit degeneracies which contribute to (33), the first-order Hamiltonian of HoloQuantum Network Theory is re-expressed as,

$$H_M^{(\mathcal{M})(1-1)}\ =\ \sum_{I}^{\text{all}}\ \mu_I\ n_I + \sum_{I\neq J}^{\text{all}}\ \lambda_J^I\ \breve{f}_J\ \mathcal{C}_J^I\ \breve{f}_I^\dagger \tag{36}$$

Being discerned in terms of all the vertonic-and-relatonic micorscopic interactions, the first-order Hamiltonian (36) is unfolded as follows,

$$\begin{aligned}H_M^{(\mathcal{M})(1-1)}\ =&\ \sum_{i=1}^{M}\ \mu_i\ n_i\ +\ \sum_{m=1}^{M}\ \sum_{i_{(m)}}^{|\{i_{(m)}\}|=\binom{M}{m}}\ \mu_{i_{(m)}} n_{i_{(m)}}\ +\\ &+\sum_{m,n=1}^{M}\ \sum_{j_{(s)},i_{(m)}}^{i_{(m)}\neq j_{(s)}}\ \lambda_{j_{(s)}}^{i_{(m)}}\ \breve{r}_{j_{(s)}} \breve{r}^\dagger_{i_{(m)}}\ +\ h.c\ +\\ &+\sum_{m=1}^{M}\ \sum_{i_{(m)}}\ \sum_{j}\ \lambda_j^{i_{(m)}}\ v_j \mathcal{C}_j^{i_{(m)}} \breve{r}^\dagger_{i_{(m)}}\ +\ h.c\ +\\ &+\sum_{j,i}^{i\neq j}\ \lambda_j^i\ v_j \mathcal{C}_j^i v_i^\dagger\ +\ h.c\end{aligned} \tag{37}$$

Results (33,36,37) conclude the mission of section three. By invoking principle nine, and directly based upon the fundamental rule, we systematically develop the total unitary evolution of the 'flavorless theory' in section four. In section five, we complete this constructive procedure, by finally presenting both the statically-and-dynamically 'perfected HoloQuantum Network Theory'.

IV. HOLOQUANTUM NETWORK THEORY: COMPLETE INTERACTIONS, THE TOTAL HAMILTONIAN, AND THE COMPACT MODEL

The mission of this section is *to complete the quantum dynamics of HoloQuantum Networks* by formulating all the higher-order microscopic interactions of the theory, and therefore, determining its total Hamiltonian which incorporates all those interaction operators. The road map to systematically get there must have already become clear to a great extent by the results of section three which formulated all the core microscopic interactions of the theory, and concluded its first-order Hamiltonian. But because to accomplish this aim, a hierarchical family of the higher-order cascade operators are to be correctly identified, it will be instructive for us to begin the procedure with carefully constructing the second-order microscopic interactions and therefore the second-order Hamiltonian of the theory.

Based on the very same principles, the higher-order many-body interactions of HoloQuantum Network Theory must be all constructed by the *'consistent prolifications' of its core operations* as being concluded by its fundamental rule. Being so, to formulate the second-order interactions, we should correctly realize the 'two-to-two' conversions between the arbitrarily-chosen 'two-sets' $\{\breve{F}_{J_1}, \breve{F}_{J_2}\} \leftrightarrow \{\breve{F}_{I_1}, \breve{F}_{I_2}\}$. Beginning with the simplest forms of these interactions, and albeit up to the redundant permutations in the down-indices and in the up-indices, we now examine the following operators,

$$\mathcal{O}^{I_1 I_2}_{J_1 J_2} \;\equiv\; (\breve{f}_{J_1} \breve{f}_{J_2})(\breve{f}^\dagger_{I_2} \breve{f}^\dagger_{I_1}) \tag{38}$$

If the interaction operators (38) are purely relatonic, namely converting any two relatons to any two relatons, then by satisfying in this simplest form all the dynamical constraints (17), they are Hamiltonian operators as such. But if any of the participating four qubits is vertonic, then the bare interactions (38) generically violate the postulated dynamical hypergraphical well-definedness of HoloQuantum Networks for the same reason explained in section three for the first-order operators. Being so, upon identifying and combining sequentially the needful first-order cascade operators for $\{\breve{F}_{J_1}, \breve{F}_{J_2}\} \leftrightarrow \{\breve{F}_{I_1}, \breve{F}_{I_2}\}$, we enlarge the bare operators (38) by the corresponding assemblies of their cascade conversions, which leads us to both of the following operators,

$$\begin{aligned}
\mathcal{O}^{(H)(I_1,I_2)}_{(2)(J_1,J_2)} &\equiv \breve{f}_{J_1} \breve{f}_{J_2} (\mathcal{C}^{I_1}_{J_1} \mathcal{C}^{I_2}_{J_2}) \breve{f}^\dagger_{I_2} \breve{f}^\dagger_{I_1} \equiv \breve{f}_{J_1} \breve{f}_{J_2}\, \mathcal{C}^{I_1 I_2}_{J_1 J_2}\; \breve{f}^\dagger_{I_2} \breve{f}^\dagger_{I_1} \\
\mathcal{O}^{(H)(I_2,I_1)}_{(2)(J_1,J_2)} &\equiv \breve{f}_{J_1} \breve{f}_{J_2} (\mathcal{C}^{I_2}_{J_1} \mathcal{C}^{I_1}_{J_2}) \breve{f}^\dagger_{I_2} \breve{f}^\dagger_{I_1} \equiv \breve{f}_{J_1} \breve{f}_{J_2}\, \mathcal{C}^{I_2 I_1}_{J_1 J_2}\; \breve{f}^\dagger_{I_2} \breve{f}^\dagger_{I_1}
\end{aligned} \tag{39}$$

By the 'descendant cascade operators' $\mathcal{C}^{II'}_{JJ'}$ defined in (39), now the operators $\mathcal{O}^{(H)(I,I')}_{(2)(J,J')}$ satisfy the constraints (17). *Result (39) concludes all the second-order cascade operators and all the second-order interactions of* $\mathcal{M}$.

Let us here highlight a point. For every arbitrarily-chosen conversion $\{\breve{F}_{J_1}, \breve{F}_{J_2}\} \leftrightarrow \{\breve{F}_{I_1}, \breve{F}_{I_2}\}$, there is a freedom in enlarging the bare operator according to either of the two distinguished channels of the one-to-one conversions which build up this process. This freedom is mirrored in the two independent second-order operators that (39) identifies. Therefore, these two independent operators must contribute to the Hamiltonian of the theory with two independent couplings, otherwise the resulted total dynamics of the theory would be restricted unnecessarily. This must be clear because the dimension of the complete space of the second-order interaction operators is counted by the basis-operators which, modulo all the redundant permutations, can be so decomposed in terms of their direct ancestors,

$$\{\; \mathcal{O}^{(H) I_1}_{J_1}\; \mathcal{O}^{(H) I_2}_{J_2} \;=\; (\breve{f}_{J_1}\, \mathcal{C}^{I_1}_{J_1}\; \breve{f}^\dagger_{I_1})\, (\breve{f}_{J_2}\, \mathcal{C}^{I_2}_{J_2}\; \breve{f}^\dagger_{I_2})\; ;\; \forall \vec{J}\;,\; \forall \vec{I}\;\} \tag{40}$$

Therefore, by the counting of (40), the Hamiltonian which generates the complete dynamics of the theory should receive an independent coupling for each one of the two operators identified in (39). By the result (39), and again by the realizition of principle eight, we obtain as follows *the second-order Hamiltonian of HoloQuantum Network Theory*,

$$H^{(\mathcal{M})(2-2)}_{M} = \sum_{J_1,J_2} \; \sum_{I_1,I_2} \;\; \lambda^{I_1 I_2}_{J_1 J_2} \;\; \breve{f}_{J_1} \breve{f}_{J_2}\, \mathcal{C}^{I_1 I_2}_{J_1 J_2}\; \breve{f}^\dagger_{I_2} \breve{f}^\dagger_{I_1} \tag{41}$$

with three structural patterns on the λ-couplings. *First*, to ensure the unitarity of the evolution,

$$\lambda^{J_1 J_2}_{I_1 I_2} = \bar{\lambda}^{I_1 I_2}_{J_1 J_2} \tag{42}$$

Second, to undo the permutational redundancies,

$$\lambda^{I_1 I_2}_{J_2 J_1} = -\lambda^{I_1 I_2}_{J_1 J_1} \quad ; \quad \lambda^{I_2 I_1}_{J_1 J_2} = -\lambda^{I_1 I_2}_{J_2 J_1} \tag{43}$$

Third, absolutely all the independent couplings $\lambda^{I_1 I_2}_{J_1 J_2}$ are the maximally-free Gaussian random parameters, to be taken from the same distributions as in (35).

By opening-up the hypergraph-state-qubit content of (39), we obtain the microscopically-distinguished second-order interactions of HoloQuantum Network Theory as follows,

$$\begin{aligned}
&\mathcal{O}^{(H)}_{(J_1,J_2),(I_1,I_2)} \;\in\; \{\; (\breve{r}_{j_{1_{(s_1)}}} \breve{r}_{j_{2_{(s_2)}}})(\breve{r}^\dagger_{i_{2_{(m_2)}}} \breve{r}^\dagger_{i_{1_{(m_1)}}})\; ;\; h.c\; ; \\
&(v_{j_1} \breve{r}_{j_{2_{(s_2)}}})\; \mathcal{C}^{i_{1_{(m_1)}}\, i_{2_{(m_2)}}}_{j_1 j_{2_{(s_2)}}}\; (\breve{r}^\dagger_{i_{2_{(m_2)}}} \breve{r}^\dagger_{i_{1_{(m_1)}}})\; ;\; h.c\; ; \\
&(v_{j_1} v_{j_2})\; \mathcal{C}^{i_{1_{(m_1)}}\, i_{2_{(m_2)}}}_{j_1 j_2}\; (\breve{r}^\dagger_{i_{2_{(m_2)}}} \breve{r}^\dagger_{i_{1_{(m_1)}}})\; ;\; h.c\; ; \\
&(v_{j_1} v_{j_2})\; \mathcal{C}^{i_{1_{(m_1)}}\, i_2}_{j_1 j_2}\; (v^\dagger_{i_2} \breve{r}^\dagger_{i_{1_{(m_1)}}})\; ;\; h.c\; ; \\
&(v_{j_1} v_{j_2})\; \mathcal{C}^{i_1 i_2}_{j_1 j_2}\; (v^\dagger_{i_2} v^\dagger_{i_1})\; ;\; h.c\; \}
\end{aligned} \tag{44}$$

Having already made explicit (44), let us manifest all the microscopically-distinct terms of (41). By taking care of all the qubit degeneracies in the summations of (41), and by appropriately renaming some of the couplings, we obtain the following unfolded form of (41),

$$
\begin{aligned}
H_M^{(\mathcal{M})(2-2)} &= \sum_{I_1,I_2} \lambda_{I_1 I_2}\ \breve{n}_{I_1}\breve{n}_{I_2}\ + \\
&+ \sum_{K}\sum_{J,I}^{J\neq I} \lambda^{K}_{J,I}\ \breve{n}_K\ (\breve{f}_J\ \mathcal{C}^I_J\ \breve{f}^\dagger_I) + \\
&+ \sum_{J_1,J_2,I_2,I_1}^{\{J_s\}\cap\{I_r\}=0} \lambda^{I_1,I_2}_{J_1 J_2}\ \breve{f}_{J_1}\breve{f}_{J_2}\,\mathcal{C}^{I_1,I_2}_{J_1,J_2}\ \breve{f}^\dagger_{I_2}\breve{f}^\dagger_{I_1}
\end{aligned}
\tag{45}
$$

By the first class of interaction terms in (45), the number operators of all possible pairs of 'the network qubits' are coupled Gaussian-randomly and 'index-globally'. By the interactions of the second term of (45) the one-to-one conversions of any two distinct qubits, coupled with the number operator of an arbitrary qubit, are turned on Gaussian-randomly. Finally, the interactions of the third class of Hamiltonian terms in (45) consist of all the possible two-to-two conversions of four distinct qubits, activated by the global and Gaussian-random couplings.

Next, the higher-order Hamiltonians of the theory must be all built up in the exact same way $H_M^{(\mathcal{M})(1-1)}$ and $H_M^{(\mathcal{M})(2-2)}$ have been constructed. Because by now all the steps are crystal clear, we summarize the final result. *The 'order-m' Hamiltonian of HoloQuantum Network Theory, $H_M^{(\mathcal{M})(m-m)}$, is concluded as follows*,

$$
H_M^{(\mathcal{M})(m-m)} = \sum_{J_1\ldots J_m}\sum_{I_1\ldots I_m} \lambda^{I_1\cdots I_m}_{J_1\cdots J_m} (\prod_{s=1}^{m}\breve{f}_{J_s})\ \mathcal{C}^{I_1\cdots I_m}_{J_1\cdots J_m} (\prod_{r=1}^{m}\breve{f}^\dagger_{I_r})
\tag{46}
$$

in which the descendant *'degree-m cascade operators'*, $\mathcal{C}^{\vec{I}}_{\vec{J}} \equiv \mathcal{C}^{I_1\ldots I_m}_{J_1\ldots J_m}$, being required to have the many-body interaction operators of $\mathcal{M}$ satisfying (17), are identified by means of their ancestors C^I_J given in (32) simply as,

$$
\mathcal{C}^{I_1\cdots I_m}_{J_1\cdots J_m} \ \equiv \ \prod_{s}^{1\leq s\leq m} \mathcal{C}^{I_s}_{J_s}
\tag{47}
$$

The couplings $\lambda^{I_1\cdots I_m}_{J_1\cdots J_m}$ in (46) are antisymmetric both in their down-indices and in their up-indices, and also for unitarity satisfy the m-body version of (42). Namely,

$$
\lambda^{I_1\cdots I_m}_{J_1\cdots J_m} = \lambda^{I_1\cdots I_m}_{[J_1\cdots J_m]} = \lambda^{[I_1\cdots I_m]}_{J_1\cdots J_m}\ ;\ \lambda^{J_1\cdots J_m}_{I_1\cdots I_m} \ = \ \bar{\lambda}^{I_1\cdots I_m}_{J_1\cdots J_m}
\tag{48}
$$

Besides (48), all the independent couplings $\lambda^{I_1\cdots I_m}_{J_1\cdots J_m}$, *at the fundamental level of the theory*, are uncorrelatedly maximally-free Gaussian-random. Namely,

$$
\mathcal{P}(\lambda^{\vec{I}}_{\vec{J}}\ |\ \mathring{\lambda}^{\vec{I}}_{\vec{J}}\,, \hat{\lambda}^{\vec{I}}_{\vec{J}}) \sim \exp\big(- \frac{\big(\lambda^{\vec{I}}_{\vec{J}} - \mathring{\lambda}^{\vec{I}}_{\vec{J}}\big)^2}{\hat{\lambda}^{\vec{I}}_{\vec{J}}}\big)
\tag{49}
$$

Now we are at *the right point to meet principle nine*.

Principle 9: $\mathcal{M}$, in being the 'it-from-qubit' theory of all quantum natures and all possible HoloQuantum Hypergraphs, must be '*covariantly complete*'.

Explanatory Note: Principle nine is cardinal to the fulfilment of the intention of HoloQuantum Network Theory, being its statement of *'covariant completeness'*. By this principle *realized on its defining 'multiverse-face'*, every 'mathematically-and-physically possible' (quantum or classical) many body system of the arbitrarily-chosen objects and their arbitrarily-chosen relations must be both completely and form-invariantly (= covariantly) formulable by HoloQuantum Network Theory. That is, the most complete time-dependent information of all the observables and all the states of all of these many body systems must be *either* extractable from the full theory as its covariant contextual model-specifications or consistent solutions, *or* be contained in its total phase diagram as the covariant effective-or-emergent theories. *Realized on its defining 'network-phase'*, this same principle states that, absolutely every dynamical hypergraphical (and so also graphical) network which is structurally and functionally consistent must be covariantly formulable from within the the full theory, either directly as extractions or solutions, or as its phase-specific effective emergences.

For principle nine to come true, given the absolute generality of our principles and all the formulation, there is *only one condition* we still need to impose. By the Wilsonian renormalization group flow, the landscape of the low-energy fixed-points of the theory must be, both physics-wise and network-wise, covariantly complete. This 'low-energy completeness' of the theory requires that, all the relevant-or-marginal multi-qubits-conversion operators which are acceptable by principles four, five, six and seven, must contribute, in the way stated by principle eight, to the total Hamiltonian defined in (12). That is, *the total Hamiltonian of $\mathcal{M}$*, namely $H_M^{(\mathcal{M})}$, *must be form-invariant under the whole Wilsonian renormalization group flow.*

Let us now realize principle nine by which the complete microscopic interactions and the total Hamiltonian of HoloQuantum Network Theory $H_M^{(\mathcal{M})}$ must be obtained. By principle two, the theory fundamentally lives in $0+1$ dimensions. Exactly in these dimensions, all the $M^\star$ hypergraph-state 'alphabet operators' $(\breve{f}_I, \breve{f}^\dagger_I)$ have canonical dimension zero, by being fermions. As such, all the acceptable convertors, no matter how 'large' they are as fermionically-made composite bosonic operators, are relevant, in fact, are 'equally relevant'. So, $H_M^{(\mathcal{M})}$ must be formed hierarchically by receiving all $\mathcal{O}^{(H)}_{(m-m)}$s. Namely, to realize the completion-principle nine, one must sums up $H_M^{(\mathcal{M})(m-m)}$, $\forall\, m^{\leq M^\star}_{\geq 1}$, to obtain $H_M^{(\mathcal{M})}$.

Therefore, *the realization of principle nine is so fulfilled*,

$$H_M^{(\mathcal{M})} = \sum_{m=1}^{m=M^\star} H_M^{(\mathcal{M})(m-m)} \tag{50}$$

Now, having realized all the nine principles, and given the individual m-body results (46,48,49), we conclude the following statement. *The complete unitary dynamics of HoloQuantum Network Theory $U_M^{(\mathcal{M})}(t)$ is generated, in accordance with* (12), *by the following total Hamiltonian*,

$$H_M^{(\mathcal{M})} = \\ = \sum_{m=1}^{m=M^\star} \{ \sum_{J_1 \ldots J_m} \sum_{I_1 \ldots I_m} \lambda^{I_1 \cdots I_m}_{J_1 \cdots J_m} \, (\prod_{s=1}^{m} \breve{f}_{J_s}) \ \ (\prod_{\ell=1}^{m} \mathcal{C}^{I_\ell}_{J_\ell}) \, (\prod_{r=1}^{m} \breve{f}^\dagger_{I_r}) \, \} \tag{51}$$

Let us make explicit the microscopic content of (51). By discerning all the index degeneracies in the summation, and upon renaming some of the the couplings, the total Hamiltonian (51) can be re-expressed as follows,

$$\begin{aligned} H_M^{(\mathcal{M})} \ = \ \sum_{m=1}^{m=M^\star} \{ & \sum_{I_r}^{1\leq r\leq m} \lambda_{I_1 \ldots I_m} \, \breve{n}_{I_1} \ldots \breve{n}_{I_m} \ + \\ & + \sum_{I_r \neq J_s}^{1\leq s,r\leq m} \lambda^{\vec{I}}_{\vec{J}} \ (\prod_{s=1}^{m} \breve{f}_{J_s}) \, \mathcal{C}^{\vec{I}}_{\vec{J}} \, (\prod_{r=1}^{m} \breve{f}^\dagger_{I_r}) \ + \\ & + \sum_{p=1}^{m} \sum_{K_c}^{1\leq c\leq p} \sum_{I_r \neq J_s}^{1\leq s,r\leq m-p} \lambda^{\vec{K}}_{\vec{J},\vec{I}} \, (\prod_{c=1}^{p} \breve{n}_{K_c}) \ \times \\ & \times \, (\prod_{s=1}^{m-p} \breve{f}_{J_s}) \, \mathcal{C}^{\vec{I}}_{\vec{J}} \, (\prod_{r=1}^{m-p} \breve{f}^\dagger_{I_r}) \, \} \end{aligned} \tag{52}$$

That is, HoloQuantum Network Theory realises three microscopically-distinguished classes of m-body hypergraph-state-qubit interactions, for every $m^{\leq M^\star}_{\geq 1}$. By the first class, (only) the number operators of m hypergraph-state-qubits interact randomly. By the second class, m one-to-one conversions between a group of all-distinguished hypergraph-state-qubits, together with their cascade conversions, are triggered randomly. By the third class, both of the above-mentioned types of interactions are simultaneously merged in arbitrary mixtures and by random strengths.

We re-highlight three important points about $H_M^{(\mathcal{M})}$. *First*, in $\mathcal{M}$, interactions between the networkical qubits are 'all-species-inclusive'. Namely, the multi-converting qubits can be all vertons, can be a number of relatons of the same-or-different degrees, or can be any arbitrary mixtures of vertons and relatons.

Second, In $\mathcal{M}$, the network qubits with all possible indices interact. That is, the defining many-body interactions are index-global.

Third, when one comes to develop application-specific sub-theories or models of $\mathcal{M}$, all the types of necessary *disciplines on the random couplings* must be imposed, by simply choosing so, by imposing extra symmetries, or by the emergences. For example, in many sub-theories, solutions or phases of the theory, the 'unfrozen' $\hat{\lambda}^{\vec{I}}_{\vec{J}}$ are reduced to only specific subsets of the network quantum spices or their indices. Moreover, the parameters of the Gaussian distributions (49) of the unfrozen couplings must be chosen, or be effectively developed, such that the characteristic statistical-average-measures of those couplings obey the necessary conditions or constraints, given the context. As a physically-significant example, in the models, solutions or phases of the theory in which *geometric locality* is either a built-in assumption or an emergent feature, usually a set of *locality constraints* on the distribution-fixing parameters of those random couplings should ensure that their associated statistical measures do properly depond on the metric-induced distances in the corresponding geometry.

Before moving to the next section, we must now feature *a remarkable sub-theory of the complete theory* (51). We will name it the *'Compactified HoloQuantum Network Theory'*, and will likewise denote it by $\mathcal{M}_{\bigcirc}$. Both structurally and qualitatively, $\mathcal{M}_{\bigcirc}$ must be regarded as *the first 'child' of HoloQuantum Network Theory* $\mathcal{M}$. $\mathcal{M}_{\bigcirc}$ as a sub-theory of $\mathcal{M}$, is remarkable by being both *'the maximal one'* and *'the minimal one'*. Being qualified as a maximal sub-theory, its definition features the following characteristics. *First*, the entirety of the hypergraph-state-qubits $\breve{F}_I$ are included in its quantum statics, so that its total Hilbert space is identical to (11). *Second*, the complete many-body interactions of HoloQuantum Network Theory are kept activated in its quantum dynamics. That is,

$$\mathcal{H}_M^{(\mathcal{M}_{\bigcirc})} \ \equiv \ \mathcal{H}_M^{(\mathcal{M})} \\ \{ \, \mathcal{O}^{(H_M^{(\mathcal{M}_{\bigcirc})})}_{(\text{order}-m)} \ ; \forall \, m^{\leq M^\star}_{\geq 1} \, \} \ \equiv \ \{ \, \mathcal{O}^{(H_M^{(\mathcal{M})})}_{(\text{order}-m)} \ ; \forall \, m^{\leq M^\star}_{\geq 1} \, \} \tag{53}$$

Being qualified at the same time as a minimal sub-theory, its definition features one more characteristics as follows. The total number of independent couplings in its total Hamiltonian of $\mathcal{M}_{\bigcirc}$ are kept as minimal as possible, with the constraint that (53) still hold. To realize that, this sub-theory is dynamized by a total Hamiltonian which exponentiates the first-order Hamiltonian (33). Namely,

$$H_M^{(\mathcal{M}_{\bigcirc})} \ \equiv \ \exp \big(H_M^{(\mathcal{M})(1-1)} \big) \ = \ \exp \big(\sum_{I,J} \lambda^I_J \, \breve{f}_J \, \mathcal{C}^I_J \, \breve{f}^\dagger_I \big) \tag{54}$$

The above Hamiltonian of $\mathcal{M}_{\bigcirc}$ which (up to the addition of an irrelevant constant term) defines a sub-theory of (51), realizes a campactification of the moduli space of the Gaussian random couplings of the complete theory $\mathcal{M}$ to that subspace which triggers (33).

V. HOLOQUANTUM NETWORK THEORY: THE MAXIMALLY-FLAVORED FORMULATION, 'THE PERFECTED THEORY'

Now, based on the same nine principles, HoloQuantum Network Theory must become 'perfected'. On one hand, this perfection amounts to accomplishing the *'absolute' mathematical generality of the theory*, by formulating the HoloQuantum hypergraphs and networks which are, both relatonically and vertonically, maximally flavored. By this ultimate enlargement of the theory, most particularly, all the consistent HoloQuantum hypergraphs and networks whose constituting quantum vertices and quantum hyperlinks are, in the most general form, *'weighted'* and also *'oriented'* are formulated most naturally. However, we must highlight, the application of the to-be-introduced flavors is not restricted to the hypergraphical weights or orientations. Most-generally understood, the maximally-falvored relatons and vertons represent all the ever-imaginable families of quantum hyperlinks and quantum vertices which, although being contextually interpreted are of different identifications, are nevertheless the constituting and interacting degrees of freedom of the 'Perfected HoloQuantum Networks'. On the other hand, this maximally-flavored formulation amounts to the *ultimate realization of principle nine in the physics-face of $\mathcal{M}$*, because the such-perfected HoloQuantum Network Theory formulates, as intended, *'absolutely' all the physically-possible many body systems of quantum objects and their quantum relations, namely, all quantum natures.*

Let us therefore begin to construct the total quantum statics, namely $\mathcal{H}_M^{(\mathcal{FM})}$, and to formulate systematically the complete quantum dynamics, namely $H_M^{(\mathcal{FM})}$, of the *'Maximally-FLavored HoloQuantum Network Theory'*. The procedure is rather straightforward formulationally, nevertheless it perfects the theory qualitatively.

First, we enlarge the so-far-built Hilbert space $\mathcal{H}_M^{(\mathcal{M})}$ be replacing its flavorless relatons with the *'upgraded relatons'* which, for every choice of the base-vertons, are *flavored in the most general form.* Clearly, networks can, and in fact do generically, develop the different 'classes = flavors' of hyperlinks which represent the distinct types of relations between the vertex-represented objects. Mathematically, for example, all possible weights or orientations of every single hyperlink can be formulated as certain flavors, as we will soon manifest. Physically, on the other hand, the maximally-flavored relatons are qubits of the 'distinct hyperlinks' which represent all the 'distinct-type' many-body interactions, many-body correlations, or the structural-or-functional many-body compositions, connections, or participatig associations. Being maximally general, *we let every relaton $R_{i_{(m)}}$ be upgraded by its most general set of the 'purely-relatonic flavors' =* $\mathrm{Set}^{R_{i_{(m)}}}\big(\alpha[i_{(m)}]\big)$, *as* $R_{i_{(m)}}^{\alpha[i_{(m)}]}$.

Second, we must likewise enlarge $\mathcal{H}_M^{(\mathcal{M})}$ by replacing its flavorless vertons with *the 'upgraded vertons' which are most-generally flavored.* By definition, the such-flavored vertons represent (as before,) information-theoretically all possible defining-or-refining states which can be associated to the arbitrarily-chosen objects, either each by each distinctly, or in the arbitrarily-chosen groups. For example, they can represent on one hand, as we will soon see, the weights of the vertices, or on the other hand, the spins, the defining 'generations', 'colors', or all sorts of defining quantum numbers of the particles. Being maximally general, therefore, *we let every single verton V_i be now upgraded by its most general set of the 'purely vertonflavors' =* $\mathrm{Set}^{V_i}\big(a_i\big)$, *as* $V_i^{a_i}$. For consistency, however, these purely-vertonic flavors do induce their 'vertonically-induced flavors' on relatons. This must be clear, because relatons are defined to be the information-theoretically-defining qubits of the quantum hyperlinks which must be uniquely identified by their now-flavored quantum base-vertices. Being so, aside from its purely-relatonic flavors, every relaton msut be additionally carrying a whole sequence of the purely-vertonic flavors which define its base-vertons, to be identified uniquely. Collecting all the data, we denote *the all-flavores-included m-relatons* by $R_{{(i^{a_i})}_{(m)}}^{\alpha[{(i^{a_i})}_{(m)}]}$.

Third, to formulate the maximally-flavored theory with notational compactness, assuming that the above index-details will be fully *remembered*, let us collectively denote the maximally-flavored hypergraph-state-qubit operators by $\big(\breve{f}_I^{\gamma_I}, \breve{f}_I^{\dagger\gamma_I}\big)$, in which $I^{\in\mathbb{N}^{\leq M^\star}}$ is the very same all-inclusive counter of the flavorless qubits. Now, *the defining total state-space of the Maximally-Flavored HoloQuantum Network Theory $\mathcal{H}_M^{(\mathcal{FM})}$ is so defined by its tensor-product structure and by its basis $\mathcal{B}_M^{(\mathcal{FM})}$*,

$$\begin{aligned}\mathcal{H}_M^{(\mathcal{FM})} &= \mathcal{H}_{(\text{vertons})}^{(\mathcal{FM})} \otimes_{1\leq m\leq M} \mathcal{H}_{(m-\text{relatons})}^{(\mathcal{FM})} \\ \mathcal{B}_M^{(\mathcal{FM})} &= \Big\{ \prod_{I_s}^{\text{all choices}} \prod_{\gamma_{I_s}}^{\text{all choices}} \breve{f}_{I_s}^{\dagger\gamma_{I_s}} \; |0\rangle \Big\}\end{aligned} \tag{55}$$

The identification (55) concludes the quantum-statical maximally-flavored realization of principles *one to three*. Reading directly from (55), the very maximally-flavored formulation of principles *seven, eight and six* must be clear. In particular, the U(1) symmetry transformations of the maximally-flavored qubits are so formulated,

$$\forall\,(I,\gamma_{I}):\; \breve{f}_I^{\gamma_I} \to e^{-i\phi}\; \breve{f}_I^{\gamma_I} \quad ; \quad \breve{f}_I^{\dagger\gamma_I} \to e^{+i\phi}\; \breve{f}_I^{\dagger\gamma_I} \tag{56}$$

Next, we move on to formulate systematically *the total unitary dynamics of the perfected theory.* Building on all the results of the previous sections, this mission must be accomplished as straightforwardly as possible. Being so, the major step to obtain $H_M^{(\mathcal{FM})}$ is to realize principles *four and five* by defining the 'maximally-flavored cascade operators', which generalize (32,47) correctly.

Remembering the notations $\{\breve{F}_I^{\gamma_I}\} = \{V_i^{a_i}; R_{(i^{a_i})_{(m)}}^{\alpha[(i^{a_i})_{(m)}]}\}$, *the maximally-flavored cascade operators* $\mathcal{C}_{J^{\gamma_J}}^{I^{\eta_I}}$*, and their order-m descendants*, are correctly identified as follows,

$$\{\ \mathcal{C}_{J^{\gamma_J}}^{I^{\eta_I}}\ \} \ \equiv\ \{\mathcal{C}_{j^{a_j}}^{I^{\eta_I}}(\mu_{j^{a_j}}^{I^{\eta_I}})\ ;\ \text{All the purely relatonic ones} = 1\}$$

$$\mathcal{C}_{j^{a_j}}^{I^{\eta_I}}(\mu_{j^{a_j}}^{I^{\eta_I}}) =$$
$$= \prod_{m=1}^{M}\ \prod_{(y^{a_y})_{(m)}}^{\text{all choices}}\ \prod_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\text{all choices}} \mathcal{C}_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{j^{a_j}\to I^{\eta_I}}\big[\mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}\big]$$

$$\mathcal{C}_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{j^{a_j}\to I^{\eta_I}}\big[\mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}\big] \ =$$
$$= \big(\ 1\ -\ \breve{n}_{(y^{a_y})_{(m)}j^{a_j}}^{\alpha[(y^{a_y})_{(m)}j^{a_j}]}\ +$$
$$+ \sum_{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}^{\text{all choices}} \mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}\ \breve{r}_{I^{\eta_I}(y^{a_y})_{(m)}}^{\dagger\beta[I^{\eta_I}(y^{a_y})_{(m)}]}\ \breve{r}_{(y^{a_y})_{(m)}j^{a_j}}^{\alpha[(y^{a_y})_{(m)}j^{a_j}]}\big)$$

$$\mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]} \ \equiv\ \bigcup_{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}^{\text{all choices}}\ \{\ \mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}\ \}$$

$$\mu_{j^{a_j}}^{I^{\eta_I}} \ \equiv\ \bigcup_{m=1}^{M}\ \bigcup_{(y^{a_y})_{(m)}}^{\text{all choices}}\ \bigcup_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\text{all choices}} \mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}$$

$$\mathcal{C}_{J_1^{\gamma_{J_1}}\ \dots\ J_m^{\gamma_{J_m}}}^{I_1^{\eta_{I_1}}\ \dots\ I_m^{\eta_{I_m}}} \ =\ \mathcal{C}_{J_1^{\gamma_{J_1}}}^{I_1^{\eta_{I_1}}}\ \cdots\ \mathcal{C}_{J_m^{\gamma_{J_m}}}^{I_m^{\eta_{I_m}}} \tag{57}$$

Here comes the identification of all the characters which participate in the generalized definitions given by (57). Every index-set $(y^{a_y})_{(m)} \equiv \{y_1^{a_{y_1}} \dots y_m^{a_{y_m}}\}$ identifies an arbitrarily-chosen set of $m_{\geq 1}^{\leq M-2}$ 'intermediate vertons' $\{V_{y_1}^{a_1}\cdots V_{y_m}^{a_m}\}$. Being highlighted, the second product in (57) incorporates the largest spectra of the flavors which must be attributed to the intermediate vertons, for them to be identified uniquely. Given the chosen $(y^{a_y})_{(m)}$, every $\breve{R}_{(y^{a_y})_{(m)}j^{a_j}}^{\alpha[(y^{a_y})_{(m)}j^{a_j}]}$ is the uniquely-identified relaton whose base indices are given by $\{j^{a_j}\}\cup(y^{a_y})_{(m)}$, and has the purely-relatonic identity-flavor 'α'. Given $j^{a_j}(y^{a_y})_{(m)}$, the third product in (57) incorporates all the 'such-based relatons' which are accommodated in the largest spectra of the purely-relatonic identity-flavors. $\breve{R}_{I^{\eta_I}(y^{a_y})_{(m)}}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}$ is, likewise, the unique relaton whose base is given by the union of $(y^{a_y})_{(m)}$ with all the vertonic indices of I^{η_I}, and is specified by the purely-relatonic identification-flavor 'β'. By the intermediate summation in (57), all the relatons with the base $I^{\eta_I}(y^{a_y})_{(m)}$ in the largest spectra of their purely-relatonic identity-flavors must be integrated.

We complete the identification of the characters in (57), by specifying the *'μ-couplings'* which, at the level of the fundamental formulation of the perfected theory, come in the definitions of its maximally-flavored cascade operators $\mathcal{C}_{j^{a_j}}^{I^{\eta_I}}(\mu_{j^{a_j}}^{I^{\eta_I}})$. Upon realizing principle eight, for every 'initial' purely-relatonic flavor $\alpha[(y^{a_y})_{(m)}j^{a_j}]$, the *'flavor-transition-couplings'* $\mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}$ in the sixth line of (57) are determined by a total number of '$|\{\beta[I^{\eta_I}(y^{a_y})_{(m)}\}| - 1$' uncorrelated *maximally-free Gaussian-random parameters* $\{\kappa_\alpha(j^{a_j};(y^{a_y})_{(m)};I^{\eta_I})\}$, ensuring that they satisfy the following constraint,

$$\sum_{\beta[I^{\eta_I}(y^{a_y})_{(m)}]}^{\text{all choices}} \mu_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[I^{\eta_I}(y^{a_y})_{(m)}]} \ =\ 1 \tag{58}$$

One can consistently restore (32) from the generalized cascade operators. The reduction is consistent because the flavorless formulation developed in sections two, three and four is equivalent with the flavored formulation in this section, by restricting to only one purely-vertonic flavor and only one purely-relatonic flavor. Because of this, upon applying the summation-constraint (58) to the one-flavor case, we obtain only one coefficient $\mu_\alpha^\alpha = 1$, such that (57) does consistently reduce to (32). Now, *for better clarity*, and as the simplest example, let us also write down here the first-order cascade operators for a 'minimally-flavored model' of HoloQuantum Networks in which vertons are flavorless, but relatons take '$\pm$' flavors. Here are the non-identity cascade operators of the model,

$$\mathcal{C}_j^{I^{\eta_I}} \ =\ \prod_{m=1}^{M}\ \prod_{y_{(m)}\equiv\{y_1\dots y_m\}}^{\text{all choices}} \mathcal{C}_{(-)}^{j\to I^{\eta_I}}[y_{(m)}]\ \mathcal{C}_{(+)}^{j\to I^{\eta_I}}[y_{(m)}]$$
$$\mathcal{C}_{(\pm)}^{j\to I^{\eta_I}}[y_{(m)}] \ =\ (\ 1 - \breve{n}_{y_{(m)}j}^{\pm}\ ) +$$
$$+ [\ \mu_\pm^-(j;y_{(m)};I^{\eta_I})\ \breve{r}_{I^{\eta_I}y_{(m)}}^{\dagger -}\ +\ \mu_\pm^+(j;y_{(m)};I^{\eta_I})\ \breve{r}_{I^{\eta_I}y_{(m)}}^{\dagger +}\ ]\ \breve{r}_{y_{(m)}j}^{\pm}$$
$$\mu_\pm^\pm(j;y_{(m)};I^{\eta_I}) = \frac{1}{2}\ \pm\ \kappa_\pm(j;y_{(m)};I^{\eta_I}) \tag{59}$$

Let us confirm that principle five is indeed realized in the perfected theory. Generators of quantum-hypergraphical isomorphisms are given by the cascade operators in (57),

$$\Gamma_{j^{a_j}}^{i^{b_i}} \ \equiv\ 1 + n_j^{a_j}(1-n_i^{b_i})[\ v_j^{a_j}\ \mathcal{C}_{j^{a_j}}^{i^{b_i}}(\hat{\mu}_{j^{a_j}}^{i^{b_i}}\ )\ v_i^{\dagger b_i} - 1\ ]$$
$$\hat{\mu}_{j^{a_j}}^{i^{b_i}} \equiv \bigcup_{m=1}^{M}\ \bigcup_{(y^{a_y})_{(m)}}^{\text{all choices}}\ \bigcup_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\text{all choices}}\ \bigcup_{\beta[i^{b_i}(y^{a_y})_{(m)}]}^{\text{all choices}} \{\ \delta_{\alpha[(y^{a_y})_{(m)}j^{a_j}]}^{\beta[i^{b_i}(y^{a_y})_{(m)}]}\ \}_{|(58)} \tag{60}$$

By (60), *the complete set of the quantum-hypergraphical isomorphisms in the perfected theory* are realized by the $\mathcal{C}_{j^{a_j}}^{i^{b_i}}$s whose flavor-transiotion-couplings are localized (by the tuned Gaussian distributions of their $\{\kappa\}$ variables) on '$\mu_\alpha^\beta = \delta_\alpha^\beta$', and in addition, they do satisfy (58).

Let us highlight the wisdom for having defined the cascade operators of the perfected theory as in (57). By the definitions in (57), not only all the enlarged conversion operators $\breve{f}_J^{\gamma_J}\mathcal{C}^{I^{\eta_I}}_{J^{\gamma_J}}\breve{f}_I^{\dagger\eta_I}$ satisfy the flavored generalizations of (17), and at the same time principle five is realized for the maxiamlly-flavored HoloQuantum Networks, but also the unitary dynamics of the perfected theory, as we now formulate in what follows, receives its most complete realization of all the nine principles.

By straghtforwardly generalizing the results of sections three and four, and utilizing the maximally-flavored cascade operators given in (57), we realize the *ninth principle* to obtain as follows *the total Hamiltonian of the Perfected HoloQuantum Network Theory,*

$$H_M^{(\mathcal{FM})} = \sum_{m=1}^{m=M^\star} H_M^{(\mathcal{FM})(m-m)}$$

$$H_M^{(\mathcal{FM})(m-m)} =$$

$$= \sum_{J_1\cdots J_m}^{\text{all}} \sum_{\gamma_{J_1}\cdots\gamma_{J_m}}^{\text{all}} \sum_{I_1\cdots I_m}^{\text{all}} \sum_{\eta_{I_1}\cdots\eta_{I_m}}^{\text{all}} \lambda^{I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}} \;\Phi^{I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}}$$

$$\Phi^{I_1^{\eta_{I_1}}\cdots I_m^{\eta_{I_m}}}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}} = \Big(\prod_{s=1}^{m} \breve{f}_{J_s}^{\gamma_{J_s}}\Big)\Big(\mathcal{C}^{I_1^{\eta_{I_1}}}_{J_1^{\gamma_{J_1}}}\cdots\,\mathcal{C}^{I_m^{\eta_{I_m}}}_{J_m^{\gamma_{J_m}}}\Big)\Big(\prod_{r=1}^{m} \breve{f}_{I_r}^{\dagger\eta_{I_r}}\Big)$$

$$\lambda^{I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}} = \lambda^{I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}}_{[J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}]} = \lambda^{[I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}]}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}}$$

$$\lambda^{I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}} = \bar{\lambda}^{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}}_{I_1^{\eta_{I_1}}\,\cdots\, I_m^{\eta_{I_m}}}$$

$$\mathcal{P}\big(\lambda^{\vec{I}^{\vec{\eta}_{\vec{I}}}}_{\vec{J}^{\vec{\gamma}_{\vec{J}}}} \mid \mathring{\lambda}^{\vec{I}^{\vec{\eta}_{\vec{I}}}}_{\vec{J}^{\vec{\gamma}_{\vec{J}}}} \,,\, \hat{\lambda}^{\vec{I}^{\vec{\eta}_{\vec{I}}}}_{\vec{J}^{\vec{\gamma}_{\vec{J}}}}\big) \;\sim\; \exp\Big(\,-\,\frac{(\lambda^{\vec{I}^{\vec{\eta}_{\vec{I}}}}_{\vec{J}^{\vec{\gamma}_{\vec{J}}}} - \mathring{\lambda}^{\vec{I}^{\vec{\eta}_{\vec{I}}}}_{\vec{J}^{\vec{\gamma}_{\vec{J}}}})^2}{\hat{\lambda}^{\vec{I}^{\vec{\eta}_{\vec{I}}}}_{\vec{J}^{\vec{\gamma}_{\vec{J}}}}}\,\Big) \tag{61}$$

By discerning all the degeneracies in the summations of (61), $H_M^{(\mathcal{FM})(m-m)}$ takes a detailed form similar to (52), but now with all the flavors being incorporated. Now, having formulated the complete unitary dynamics of the perfected theory, the *'Compactified Maximally-Flavored HoloQuantum Network Theory'* must be also formulated, similar to its flavourless counterpart defined in section four. This is the 'minimal-maximal' sub-theory of the perfected theory, with the same quantum statics as (55), but with the total hamiltonian which exponentiates the first-degree Hamiltonian in (61). Namely,

$$\mathcal{H}_M^{(\mathcal{FM}\bigcirc)} \;=\; \mathcal{H}_M^{(\mathcal{FM})}$$

$$\{\,\mathcal{O}_{(m)}^{(H)(\mathcal{FM}\bigcirc)}\;;\;\forall\, m_{\geq 1}^{\leq M^\star}\,\} \;=\; \{\,\Phi^{I_1^{\eta_{I_1}}\cdots I_m^{\eta_{I_m}}}_{J_1^{\gamma_{J_1}}\,\cdots\, J_m^{\gamma_{J_m}}}\;;\;\forall\, m_{\geq 1}^{\leq M^\star}\,\}$$

$$H_M^{(\mathcal{FM}\bigcirc)} = \exp\Big(\sum_{J\,,\,\gamma_J}^{\text{all}}\;\sum_{I\,,\,\eta_I}^{\text{all}}\;\lambda^{I^{\eta_I}}_{J^{\gamma_J}}\;\breve{f}_J^{\gamma_J}\;\mathcal{C}^{I^{\eta_I}}_{J^{\gamma_J}}\;\;\breve{f}_I^{\dagger\eta_I}\Big) \tag{62}$$

The perfected theory, which we have already concluded, has been axiomatically defined and formulated based on the nine principles. Most remarkably, its total Hilbert space (55) and its total unitary dynamics (61) are by their axiomatic constructions the most general, the most fundamental and the most complete. By the acomplished ultimate realization of principle nine, the perfected theory is covariantly complete. Being so, even sheerly regarded as a theory of quantum mathematics, the Maximally-Flavored HoloQuantum Network Theory is the most-complete theory of *'all possible forms' of dynamical matematically-quantized hypergraphs*. Now, given the extremely-generic domain of all the dynamical networks which are formulated by the flavorless theory, defined in the first four sections, we must take in what follows one and only one more step to realize the 'hypergraphical covariant completeness' of the maximally-flavored theory. That is, we must work-out how the perfected theory formulates the dynamical mathematically-quantized hypergraphs which are defined with *arbitrarily-oriented hyperlinks*, and with *arbitrarily-weighted vertices and hyperlinks.*

The most natural direct way that one accomplishes the complete formulation of the maximally-oriented and maximally-weighted quantized hypergraphs is to extract it as a sub-theory of (55) and (61), upon defining all the orientations and all the weights as specific types of flavors. Let us state the central idea. *In HoloQuantum Network Theory, every relaton of every arbitrarily-chosen orientation or weight, and every verton of every arbitrarily-chosen weight, must be distinctively taken to be one independent quantum degree of freedom of* (55)*, that is, it should be identified with one correspondingly-flavored fermion-qubit* $F_I^{\gamma_I}$. Being so, the promised theory of the maximally-oriented and maximally-weighted HoloQuantum Networks can remain purely fermionic, and one does not need any bosonic degrees of freedom to 'weight stuff' or to 'orient stuff'. Restated, the set of all the relatonic orientations, and the sets of all the vertonic weights and all the relatonic weights, must be mapped in one-to-one manners to their corresponding sets of the network-qubit flavors.

Let us now begin to formulate the above-featured sub-theory of the perfected theory with identifying as such all the hyperlink orientations in a HoloQuantum Network. By definition, a hyperlink of m base-vertices has $m!$ orientations, each one corresponding to a unique ordering of its base-vertices. As such, to formulate the most general orientations of the quantum hyperlinks, every m-relaton, identified earlier in (11) as $\breve{R}_{i_{(m)}}$, must be now 'branched-out' as the $m!$ 'orientationally-flavored relatons' $\breve{R}_{i_{(m)}}^{\alpha_{(m)}}$ in the Hilbert space (55). Namely, *the maximally-oriented HoloQuantum Hypergraphs* must be formulated by this family of *orientation flavors*,

$$\forall\; i_{(m)} \quad,\quad \breve{R}_{i_{(m)}} \quad\hookrightarrow\quad \{\;\breve{R}_{i_{(m)}}^{\alpha_{(m)}}\;\;;\;\alpha_{(m)} \in \mathbb{N}^{\leq m!}\;\} \tag{63}$$

Like orientations (63), all the *hypergraphical weights* form a distinct set of the flavors, named the '*weight flavors*'. Stating precisely, the 'all-inclusive set' of all the defined weights for the quantum constituents of a HoloQuantum Network, being the vertices or the m-degree hyperlinks, is mapped in a one-to-one manner to a complete set of vertonic-and-relatonic weight flavors. Being maximally general, the weight flavor of every structural-qubit F_I comes with its distinct labeling $\omega(F_I) \equiv \omega_I$, and also with a distinct arbitrarily-chosen spectrum $\mathcal{W}_I \equiv \{\omega_I\}$. That is every hypergraph-state-qubit F_I is branched-out by taking flavors as follows,

$$\forall\, I \quad ; \quad F_I \quad \hookrightarrow \quad \{\, F_I^{\,\omega_I} \quad ; \quad \omega_I \in \mathcal{W}_I \} \quad ; \tag{64}$$

Therefore, the most general quantum statics and unitary quantum dynamics of the HoloQuantum Networks which are *both maximally oriented and maximally weighted* are given by the Hilbert space (55) and the total Hamiltonian (61) whose *'structural flavors'* are so identified,

$$\{\, F_I^{\gamma_I} \,\} \;\equiv\; \{\, V_i^{\omega_i} \quad ; \quad R^{\alpha_{(m)};\omega[(i^\omega)_{(m)}]}_{(i^\omega)_{(m)}} \,\} \tag{65}$$

We highlight a point about the many-body interactions (61) of the HoloQuantum Network Theory endowed with the structural flavors (65). This point is a direct implication of the maximally-flavored version of the fundamental rule, which is equal to the very same rule stated in section three, upon the replacements $F_I \hookrightarrow F_I^{\gamma_I}$ and $\mathcal{C}_J^I \hookrightarrow \mathcal{C}_{J^{\gamma_I}}^{I^{\gamma_I}}$. Given this rule, at the level of the fundamental formulation of the theory, not only the quantum vertices and all the quantum hyperlinks of different degrees can freely convert into one another, but also now all their different orientations and all their different weights can convert into one another.

The triplet (55,61,65) does conclude the promised mathematical covariant-completion of the flavorless 'basic-class' theory, by defining and formulating the HoloQuantum Networks which are endowed with the most general orientations and the most general weights. That is, the most complete purely-information-theoretic formulation of the most general family of dynamical mathematically-quantized hypergraphs is provided by the above triplet.

However, for bringing a good example of HoloQuantum Networks in which some flavors take continuous spectra, and also because weights are usually taken to be real or complex numbers, *let us now treat the spectra of all the weight-flavors to be the arbitrarily-chosen continues sets.* By this, the operators of the hypergraph-state-qubits are turned into those of the fermionic field-functions which live on the topological product of the time-dimension and an abstract 'wight-space'. Being so, one, formally obtains an example of the *'HoloQuantum-Networkical Continuum Quantum Field Theory'* whose statics is purely fermionic, and whose dynamics is sourced by all the many-body interactions we have already defined.

Being so, let us now formulate even-handedly the hypergraph-state-qubits which are, as already specified, both maximally oriented and maximally weighted by $\breve{f}_I^{(\dagger)\alpha_I}(\vec{x}_I \equiv \vec{\omega}_I)$,

$$\{\breve{f}_I^{(\dagger)\alpha_I}(\vec{x}_I)\} = \{\; v_i^{(\dagger)}(\vec{x}_i) \;;\; r^{(\dagger)\alpha_{(m)}}_{i_{(m)}}(\vec{x}_{i_{(m)}} \mid \vec{x}_1, \cdots, \vec{x}_m) \;\} \tag{66}$$

In the right hand side of (66), each finite-dimensional vector $\vec{x}_I$ identifies the continuously-valued weight of the corresponding hypergraph-state-qubit F_I, whose spectrum can be independently chosen, to be maximally general. For example, if the corresponding weight is a real number, $\vec{x} \equiv x^{\,\in\mathbb{R}}$, but if it is a complex number, $\vec{x} \;\equiv\; (z, \bar{z})^{\,\in\Sigma}$ with Σ being the complex plain $\mathbb{C}$ or a Riemannian surface whose topology one chooses arbitrarily. So, in the models where all the weights are real numbers, or complex numbers, we formally get a HoloQuantum-Hypergraph-Networkical purely-fermionic novelly-interacting continuum quantum field theory living on the Lorentzian manifolds similar to the $(1+1)$ dimensional or the $(2+1)$ dimensional spacetimes. To highlight, HoloQuantum networks with multiple hyperlinks correspond to the special case in which relatons are integrally weighted, namely $\vec{x} = n^{\in\mathbb{N}}$.

The cascade operators $\mathcal{C}^{I^{\beta_I}}_{J^{\alpha_J}}(\vec{x}_J;\vec{x}_I)$ and the total Hmailtonian of the complete unitary dynamics of this continuously-flavored sub-theory are defined as in (57,61), upon turning the summations and the products over the continuous weights into integrations and the exponentials of logarithmic integrations, respectively. In particular, its *compactified sub-theory* is defined by,

$$H_M^{(\mathcal{FM}_\bigcirc)} = \exp\Big(\sum_{J,I}\sum_{\alpha_J,\beta_I}\int d\vec{x}_I d\vec{x}_J\; \lambda^{I^{\beta_I}}_{J^{\alpha_J}}(\vec{x}_J;\vec{x}_I)\; \times \\ \times\; \breve{f}_J^{\alpha_J}(\vec{x}_J)\, \mathcal{C}^{I^{\beta_I}}_{J^{\alpha_J}}(\vec{x}_J;\vec{x}_I)\; \breve{f}_I^{\dagger\beta_I}(\vec{x}_I)\,\Big) \tag{67}$$

Results (55,61) conclude the 'Perfected HoloQuantum Network Theory', axiomatically built upon all the nine principles. As promised in the beginning of this work, the perfected theory is the most fundamental and the most complete covariant 'it-from-qubit' theory of both all the quantized networks and all quantum natures.

VI. HOLOQUANTUM NETWORK THEORY: MODELS WITH EXTRA SYMMETRIES, SUPERSYMMETRIC EXAMPLES

Because HoloQuantum Network Theory is built to be the quantum many body theory of all physically-possible systems of quantum objects and quantum relations, it *must be necessarily maximally minimalistic in taking its fundamental symmetries*. This minimalism is implied by the nine principles, so that the only symmetries transformations in $\mathcal{M}$ are *'the unavoidable ones'*.

Being minimalistic, the perfected theory only has the $U(1)$ symmetry for the global-phase redundancies, the minimally-broken symmetry of qubits-equal-treatment, and moreover has received in it the complete set of quantum-hypergraphical isomorohisms realized by only some of its exact many-body-interaction operators. On the other hand, in the application-specific sub-theories and models which are derived from within the perfected theory, or in its contextually-motivated solutions or phases, generically some types of '*extra symmetries*' either must be imposed or must be emerged. Let us highlight in here two highly-motivated classes of such extra symmetries. Because hypergraphs are, at the level of their fundamental definition, pregeometric and background-less, indeed there can be no spatially-local fundamental symmetries, and no fundamental gauge symmetries in the perfected theory. However, in both the *'emergent-spacetime'* and the *'standard-model-like'* sub-theories or phases of the perfected theory, a number of gauge symmetries must manifest as such types of extra symmetries.

As an instructive and also interesting illustration of this point, here we will incorporate *supersymmetry* as one such *extra symmetry* in a maximal sub-theory of the perfected theory. As it indeed must be, it will be demonstrated that to formulate the most general family of supersymmetric HoloQuantum Networks, one needs neither to introduce any extra degrees of freedom into the total quantum statics, nor to deform the complete total quantum dynamics of the perfected theory. That is, the perfected theory stays statically and dynamically form-invariant under this extra-symmetry imposition, although indeed some of its otherwise-free structures will be constrained. We formulate here two different model-theories of HoloQuantum Networks which are endowed with $\mathcal{N}=2$ global supersymmetries. Moreover, to illustrate the simplest yet sufficiently-rich examples, we suffice to the symmersymmetrization of the flavorless purely-relatonic HoloQuantum Networks. The work-out recipes are straightforward, yielding supersymmetric HoloQuantum Networks which are fundamentally purely-fermionic, and dynamized with the Hamiltonians which are embedded in (51).

One therefore introduces a complex-conjugate pair of supercharges $(Q_M^{(\mathcal{M}_{(s)})}, \bar{Q}_M^{(\mathcal{M}_{(s)})})$ in the all-fermionic Hilbert space of HoloQuantum Network Theory $\mathcal{H}_M^{(\mathcal{M})}$ whicap every hypergraph-state fermion qubit $\breve{f}_I^{(\dagger)}$ to a *fermion-made composite* bosonic qubit $\breve{b}_I^{(\dagger)}$ as its supersymmetric partner, and vice versa. The algebra is,

$$\begin{aligned}
&(Q_M^{(\mathcal{M}_{(s)})})^2 = (\bar{Q}_M^{(\mathcal{M}_{(s)})})^2 = 0 \\
&[Q_M^{(\mathcal{M}_{(s)})}, \breve{f}_I^\dagger] = \breve{b}_I^\dagger \ ; \ [Q_M^{(\mathcal{M}_{(s)})}, \breve{f}_I] = 0 \ ; \ h.c \\
&[Q_M^{(\mathcal{M}_{(s)})}, \breve{b}_I] = \partial_t \breve{f}_I \ ; \ [Q_M^{(\mathcal{M}_{(s)})}, \breve{b}_I^\dagger] = 0 \ ; \ h.c
\end{aligned} \tag{68}$$

We formulate*the most general family of supersymmetruc HoloQuantum Networks*. Being so, the supersymmetric charges $(Q_M^{(\mathcal{M}_{(s)})}, \bar{Q}_M^{(\mathcal{M}_{(s)})})$ must be the largest-possible Gaussian-random composite fermionic fields, being made from the hypergraph-state qubits $\{\breve{f}_J\}$, $\{\breve{f}_I^\dagger\}$, whose supersymmetric Hamiltonian, $H_M^{(\mathcal{M}_{(s)})}$,

$$H_M^{(\mathcal{M}_{(s)})} \equiv \{Q_M^{(\mathcal{M}_{(s)})}, \bar{Q}_M^{(\mathcal{M}_{(s)})}\} \tag{69}$$

satisfies the *'maximal-dynamical-embedding'* criterion,

$$H_M^{(\mathcal{M}_{(s)})} \subset^{(\text{embedded maximally})} \mathcal{H}_M^{(\mathcal{M})} \tag{70}$$

The first supersymmetrization recipe is inspired by the initiative work [3], being the best method of realizing *a supersymmetric sub-theory of $\mathcal{M}$ which is maximal both statically and dynamically.* By this recipe, one imposes an entirely-arbitrary $\mathbb{Z}_2$ partitioning on the total Hilbet space $\mathcal{H}_M^{(\mathcal{M})}$. By this imposition, the hypergraph-state fermion qubits are partitioned into two 'chirality classes',

$$\begin{aligned}
&\mathbb{Z}_2 \ : \ \psi^{(\dagger)}_{\hat{I}(-)} \longleftrightarrow \psi^{(\dagger)}_{\hat{I}(+)} \\
&\{ \ \breve{f}_I^{(\dagger)} \ \} \ \equiv \ \{ \ \psi^{(\dagger)}_{\hat{I}(-)} \ ; \ \psi^{(\dagger)}_{\hat{I}(+)} \ \}
\end{aligned} \tag{71}$$

Let us highlight that the choice of this $\mathbb{Z}_2$ partitioning is absolutely arbitrary, and so all the different choices for it results in the supersymmetric models which may differ interpretationally, but indeed as supersymmetric quantum theories are all physically equivalent. Next, in accordance to the chirality partitioning (71) imposed on the hypergraph-state-qubits $\breve{F}_I$, the two supersymmetric charges of the sub-theory $\mathcal{M}_{(s)}$ are so defined,

$$\begin{aligned}
Q_M^{(\mathcal{M}_{(s)})} \equiv & \sum_{\hat{J}_1}^{\text{all}} \eta_{\hat{J}_1} \psi_{\hat{J}_1(+)} + \sum_{\hat{J}_1 \hat{J}_2 \hat{I}_1}^{\text{all}} \eta_{\hat{J}_1 \hat{J}_2 \hat{I}_1} \psi_{\hat{J}_1(+)} \psi_{\hat{J}_2(+)} \psi^\dagger_{\hat{I}_1(-)} + \ldots = \\
&= \sum_{m=0}^{\frac{M^\star}{2}-1} \sum_{\hat{J}_{(m+1)}}^{\text{all}} \sum_{\hat{I}_{(m)}}^{\text{all}} \eta_{\hat{J}_1 \cdots \hat{J}_{m+1} \hat{I}_1 \cdots \hat{I}_m} \times \\
&\times \ (\psi_{\hat{J}_1(+)} \cdots \psi_{\hat{J}_{(m+1)}(+)}) \, (\psi^\dagger_{\hat{I}_m(-)} \cdots \psi^\dagger_{\hat{I}_1(-)}) \\
\bar{Q}_M^{(\mathcal{M}_{(s)})} \equiv & \sum_{\hat{I}_1}^{\text{all}} \bar{\eta}_{\hat{I}_1} \ \psi^\dagger_{\hat{I}_1(+)} + \sum_{\hat{J}_1 \hat{I}_1 \hat{I}_2}^{\text{all}} \bar{\eta}_{\hat{J}_1 \hat{I}_1 \hat{I}_2} \ \psi_{\hat{J}_1(-)} \psi^\dagger_{\hat{I}_2(+)} \psi^\dagger_{\hat{I}_1(+)} + \ldots = \\
&= \sum_{m=0}^{\frac{M^\star}{2}-1} \sum_{\hat{J}_{(m)}}^{\text{all}} \sum_{\hat{I}_{(m+1)}}^{\text{all}} \bar{\eta}_{\hat{J}_1 \cdots \hat{J}_m \hat{I}_1 \cdots \hat{I}_{m+1}} \times \\
&\times \ (\psi_{\hat{J}_1(-)} \cdots \psi_{\hat{J}_m(-)}) \, (\psi^\dagger_{\hat{I}_{(m+1)}(+)} \cdots \psi^\dagger_{\hat{I}_1(+)})
\end{aligned} \tag{72}$$

In the definitions (72), all the independent defining couplings $\eta_{\hat{J}_1 \cdots \hat{J}_{m+1}; \hat{I}_1 \cdots \hat{I}_m}$s are the statistically uncorrelated Gaussian random parameters being selected the same distributions (49) for the λ-couplings of $\mathcal{M}$.

By selecting the supersymmetric charges to be the ones constructed in (72), as we can concretely compute, one realizes the the maximal sub-theory of 'supersymmetric HoloQuantum Networks', $\mathcal{M}_{(s)}$. That is, the required dynamical maximality criterion (70) is satisfied by the resulted supersymmetric total Hamltonian (69), in which a family of *'composite-random couplings'* is employed,

$$(\lambda^{I_1 \cdots I_m}_{J_1 \cdots J_m} \; ; \; \bar{\lambda}^{I_1 \cdots I_m}_{J_1 \cdots J_m}) = \\ = \text{ some c.c functions of } \{ \, \eta_{\hat{J}_1 \cdots \hat{J}_s \hat{I}_1 \cdots \hat{I}_{s+1}} \, , \, \bar{\eta}_{\hat{J}_1 \cdots \hat{J}_r \hat{I}_1 \cdots \hat{I}_{r+1}} \, \} \tag{73}$$

We highlight that such a '($\pm$) partitioning' of the fermionic Hilbert space is very natural in many of the sub-theories and models which are derived from within $\mathcal{M}$, an example of which being the Wheelerian toy model presented in the next section. Being so, such models are all naturally amanable to the very same supersymmetrization we have presented here, albeit whenever their model-defining constraints respect supersymmetry.

Let us here mention an alternative recipe. Similar to the supersymmetrization in [4], the supercharges can be also defined as follows,

$$\begin{aligned} Q^{(\mathcal{M}_{(s)})}_{M} \; &\equiv \; \sum_{m=1}^{\frac{M^\star}{2}} q_{(2m-1)} \sum_{\text{all } Js}^{1 \leq s \leq m} \eta_{J_1 \cdots J_{2m-1}} \breve{f}_{J_1} \cdots \breve{f}_{J_{2m-1}} \equiv \\ &\equiv \sum_{m=1}^{\frac{M^\star}{2}} q_{(2m-1)} \, Q^{(\mathcal{M}_{(s)})}_{M(2m-1)} \quad ; \quad \bar{Q}^{(\mathcal{M}_{(s)})}_{M} \equiv (Q^{(\mathcal{M}_{(s)})}_{M})^\dagger \end{aligned} \tag{74}$$

with the $q_{(2m-1)}$s being formal complex coefficients, by which all the 'fixed-order supercharges' $Q_{M(2m-1)}$ are distinguished. By taking the supercharges given in (74), the resulted supersymmetric Hamiltonian (69) develops, in addition to the resulted embedding of $H^{(\mathcal{M})}_{M}[\{\lambda^{\vec{I}}_{\vec{J}}\}, c.c, \}]$, the following terms ΔH_M,

$$\begin{aligned} \Delta H_M = \sum_{0 \leq m,n \leq \frac{M^\star}{2}}^{(n \neq m)} \; \sum_{\vec{J}_{(2m)}, \vec{I}_{(2n)}}^{\text{all possible}} \lambda^{\vec{I}_{(2n)}}_{\vec{J}_{(2m)}} (\prod_{L}^{\text{some}} \breve{n}_L)(\prod_{s=0}^{2m} \breve{f}_{J_s})(\prod_{r=0}^{2n} \breve{f}^\dagger_{I_r}) \\ \lambda^{\vec{I}_{(2n \neq 2m)}}_{\vec{J}_{(2m)}} \; \equiv q_{(2m+1)} \bar{q}_{(2n+1)} \sum_{K}^{M^\star} \eta_{K \vec{J}_{(2m)}} \bar{\eta}_{K \vec{I}_{(2n \neq 2m)}} \end{aligned} \tag{75}$$

However, these extra terms do violate the global $U(1)$ symmetry of HoloQuantum Network Theory, which is necessary for its well-definedness as a background-less quantum theory. By demanding the preservation of the $U(1)$ symmetry, $\Delta H_M = 0$, we get a set of algebraic constraints on the η parameters, whose generic solution allows only a single q-coefficient to be non-zero. That is, we will get a sub-theory with only a pair of 'fixed-order supercharges' $(Q^{(\mathcal{M}_{(s)})}_{M(2m-1)} \, ; h.c)$. As such, we favor the first supersymmetrization recipe in light of the principle nine of $\mathcal{M}$.

VII. HOLOQUANTUM NETWORK THEORY: WORKING OUT A SIMPLEST TOY MODEL OF 'WHEELERIAN PARTICIPATORY UNIVERSE'

In this section, from within HoloQuantum Network Theory, we will work out a concrete 'simplest toy model' on two purposes. *First*, totally independent of the specific physics context being presented here, we want to exemplify how concrete models of phenomenological interests can be extracted-out from the quantum statics and the unitary quantum dynamics of the total theory. *Second*, we present a minimalistic simplest toy model of the *'HoloQuantum-Networkically-Realized Wheelerian Universe'*. By Wheelerian Universe, we mean a quantum universe which is constructed upon, and so realizes, *all the three principal visions of Wheeler* [1]. These three principles are so stated. *First*, the quantum universe as a whole is a quantum network of *'many-observer participencies'*, the actions of which are identified with her 'elementary quantum-phenomena'. *Second*, there is a most-fundamental statistical randomness underlying this quantum universe which sources the *'law(s) without law(s)'*. *Third*, the principle of *'it from (qu)bit'* states that all the physical world is composed of, and assembled by, all the fundamentally-random 'answering-qubits' of 'yes-or-no's to the abundantly-many binary questions posed by all the participatory-observers. We emphasize that the manners in which these three principles are incorporated in the toy model here are intentionally set to be the most minimalistic, the simplest, and mainly instructive. But, here comes the one very significant massage that we want to convey here. *The above three Wheelerian principles can be correctly merged and realized only by the theory $\mathcal{M}$ which has been defined, formulated and perfected in this work. In fact, the yes-or-no qubits of the participatory universe must be identified with the very network qubits which formulate information-theoretically the HoloQuantum Hypergraphs, and so microscopically interact with one another as given by the total Hamiltonian of the perfected theory* (61).

Let us now conceive and formulate step-by-step the simplest toy model of a Wheelerian participancy universe which is 'purely-relational'. By being purely relational, we mean the following simplification in this minimal toy model. We will take all of the participatory observes to be fixedly frozen in their states of presence, and then let the relatonic qubits of the 'multi-observer participany relations' be the only dynamical quantum degrees of freedom. It is clear that in the ultimate theory of the Wheelerian universe, every participatory observer must also be quantumly active as a dynamical verton which can switch between the states of absence and presence. This implies that all the nontrivial cascade operators should be necessarily in role in any realistic Wheelerian model. However, in here we reduce the today model to be purely relational, on the account of the first purpose of this section which is instructional.

By this explanation, let us consider a total number of M participatory observers who are all fixedly present. To formulate their Wheelerian Participatory Universe from within $\mathcal{M}$, let us employ the total Hilbert space $\mathcal{H}_M^{(\mathcal{FM})}$ (55) of M flavorless vertons together with all their 'evenly-flavored' m-relatons. The vertons of $\mathcal{H}_M^{(\mathcal{FM})}$ represent, by any one-to-one correspondence, those very M quantum participatory observers. Therefore, by assumption, all the vertons V_i must stay frozen in their presence-states, in a dynamically consistent manner. Because the vertons are fixedly present, we have the first set of the quantum constraints of the toy model by the following vertonic operator identities,

$$n_i = 1 \quad , \quad \forall\, i \in \mathbb{N}^{\leq M} \tag{76}$$

Wheelerianly, *the fundamental degrees of freedom* which microscopically *assemble* this whole quantum universe, are *all the randomly-sourced 'yes-or-no' answering-qubits to 'a complete set' of questions posed by, and shared by, all the subsets of all the M participatory-observers.* The completeness of the set of questions means that if we were given the qubit data of only any proper subset of those binary questions, then the participatory universe could not have been assembled by them entirely. Now, let every complete set of these yes-or-no answering-qubits be collected in the set of doublets $\{(\mathcal{Y}_{i_{(m)}}^{\alpha_{i_{(m)}}}\ ,\ \mathcal{N}_{i_{(m)}}^{\alpha_{i_{(m)}}})\}$, in which $i_{(m)}$ identifies the corresponding choice of $m^{\leq M}$ participatory questioners, while the index $a_{i_{(m)}}$ forms *a complete spectrum* of the binary questions posed by those same observers. The simplest HoloQuantum Networkian toy-modeling of this Wheelerian scenario is 'all relatonic'. Therefore, the hypergraph-state relatons should not only carry the very same question-spectrum flavors, but also be further doubly-flavored, as follows,

$$\begin{aligned}
&\{\ \breve{R}_{i_{(m)}}^{\alpha_{i_{(m)}},\nu}\ \} \ \equiv\ \{\ \breve{R}_{i_{(m)}}^{\alpha_{i_{(m)}},\pm}\ \}\\
&\{\ \breve{R}_{i_{(m)}}^{\alpha_{i_{(m)}},-}\ \} \longleftrightarrow \{\ \mathcal{N}_{i_{(m)}}^{\alpha_{i_{(m)}}}\ \}\\
&\{\ \breve{R}_{i_{(m)}}^{\alpha_{i_{(m)}},+}\ \} \longleftrightarrow \{\ \mathcal{Y}_{i_{(m)}}^{\alpha_{i_{(m)}}}\ \}
\end{aligned} \tag{77}$$

By (77), every such-identified degree-m relaton whose chirality flavor is positive (negative) represents the positive (negative) answer given to one of the binary questions posed by those m participatory observers who are in correspondence with its base-vertons. Let us highlight that, by (76), the hypergraph-state-relatons $\breve{R}_{i_{(m)}}^{\alpha_{i_{(m)}},\nu}$ are operationally equal to their counterprarts $R_{i_{(m)}}^{\alpha_{i_{(m)}},\nu}$, everywhere in this simplest toy model.

Relatons (77) are the active quantum degrees of freedom which assemble the Wheelerian Universe. Namely, the operators $\big(\breve{r}_{i_{(m)}}^{\alpha_{i_{(m)}},\pm}, \breve{r}_{i_{(m)}}^{\dagger\alpha_{i_{(m)}},\pm}\big)$ are set free to annihilate and create their yes-or-no answering-qubits, and therefore, *to re-assemble the universe.*

Because of the information-theoretic binary nature of every pair of relatons with opposite chiralities, one must impose one more set of quantum constraints. *The defining basis-states of the total Hilbert space must be mapped, in a one-to-one manner, to the outcomes of the 'all-measurements' which correspond to the 'all-questions-answered' entirely-assembled Wheelerian universes of these M participatory observers.* That is, the evolving global wavefunction of the Wheelerian universe must be, at any arbitrary time, a superposition of the basis-states in which, for every choice of $m^{\leq M}$ observers $i_{(m)}$, and for every of the items $\alpha_{i_{(m)}}$ in their questionnaire, one of the two $R_{i_{(m)}}^{\alpha_{i_{(m)}},\pm}$ relatons is 'present', whereas its chirality-complement one is necessarily 'absent'. Formulated as operator identities, we must therefore demand the following second set of quantum constraints as relatonic operator identities,

$$\breve{n}_{i_{(m)}}^{\alpha_{i_{(m)}},-} + \breve{n}_{i_{(m)}}^{\alpha_{i_{(m)}},+} = 1 \quad , \quad \forall\, i_{(m_{\leq M})}\ ,\ \forall\, \alpha_{i_{(m_{\leq M})}} \tag{78}$$

By imposing (76) and (78), the total Hilbert space of the simplest toy model of the *Wheelerian Participatory Universe of the M observers*, $\mathcal{H}_M^{(\mathcal{W.P.U})}$, is given by the truncation of $\mathcal{H}_M^{(\mathcal{FM})}$ whose defining basis is identified as follows,

$$\begin{aligned}
\mathcal{B}_M^{(\mathcal{W.P.U})} \ = \ \{ \prod_{m=1}^{M} \prod_{i_{(m)}}^{\text{all}} \prod_{\alpha_{i_{(m)}}}^{\text{all}} \ \big[\ (1-\epsilon_{i_{(m)}}^{\alpha_{i_{(m)}}})\ \breve{r}_{i_{(m)}}^{\dagger\alpha_{i_{(m)}},-} \ +&\\
+\ \epsilon_{i_{(m)}}^{\alpha_{i_{(m)}}}\ \breve{r}_{i_{(m)}}^{\dagger\alpha_{i_{(m)}},+}\ \big]\ |0\rangle&\\
,\ \text{for all possible choices of } \epsilon_{i_{(m_{\leq \hat{M}})}}^{\alpha_{i_{(m)}}} \in \{0,1\}\ \}&
\end{aligned} \tag{79}$$

Moreover, the quantum-statical truncation (79), which in turn is demanded by the quantum constraints (76,78), must be dynamically consistent. That is, clearly, the truncation defined above must be always preserved by the evolution of the so-defined HoloQuantum Networks. For this dynamical consistency to come true, the total Hamiltonian of the Wheelerian Participatory Universe which generates its unitary evolution, $H_M^{(\mathcal{W.P.U})}$, must satisfy the following costraints,

$$\begin{aligned}
&[\ H_M^{(\mathcal{W.P.U})}\ ,\ n_i\ ]\ =\ 0 \quad , \quad \forall\, i_{\leq M}\\
&[\ H_M^{(\mathcal{W.P.U})}\ ,\ \breve{n}_{i_{(m)}}^{\alpha_{i_{(m)}},-} + \breve{n}_{i_{(m)}}^{\alpha_{i_{(m)}},+}\ ]\ =\ 0 \quad , \quad \forall\, i_{(m_{\leq M})}\ ,\ \forall\, \alpha_{i_{(m)}}
\end{aligned} \tag{80}$$

Being promised from the very beginning, the above total Hamiltonian $H_M^{(\mathcal{W.P.U})}$ must be *directly* extracted from the total Hamiltonian of the perfected HoloQuantum Network Theory (61). To explicitly fulfill this demand, let us take the first-degree Hamiltonian of the perfected theory, manifested in the exponent of (62), and have it now expressed as the so-flavored cousin of (36).

By applying (80,76,78) to that, and dropping constant terms, we present as follows the *first-degree Hamiltonian of the 'most minimalistic' toy model of the HoloQuantum Networkical Wheelerian Participatory Universe*,

$$H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})(1-1)} = \sum_{\text{all } i_{(m)}}^{1\le m\le M} \sum_{\text{all } \alpha_{i_{(m)}}} \Big(\mu_{i_{(m)}}^{\alpha_{i_{(m)}},+} \breve{n}_{i_{(m)}}^{\alpha_{i_{(m)}},+} + \bar{\lambda}_{i_{(m)}}^{\alpha_{i_{(m)}},-+} \breve{r}_{i_{(m)}}^{\alpha_{i_{(m)}},+} \breve{r}_{i_{(m)}}^{\dagger\alpha_{i_{(m)}},-} + \lambda_{i_{(m)}}^{\alpha_{i_{(m)}},-+} \breve{r}_{i_{(m)}}^{\alpha_{i_{(m)}},-} \breve{r}_{i_{(m)}}^{\dagger\alpha_{i_{(m)}},+} \Big) \tag{81}$$

Let us highlight that, because in this simplest model of Wheelerian HoloQuantum Network, all the vertons are present fixedly by the imposed quantum constraint (76), the nontrivial cascade operators do not appears in the Hamiltonian. To remind, all the independent $\mu_{i_{(m)}}^{\alpha_{i_{(m)}},+}$ and the $\lambda_{i_{(m)}}^{\alpha_{i_{(m)}},-+}$ couplings in (81) must be taken to be uncorrelated maximally-free Gaussian-random as in (61). The above first-degree Hamiltonian can be more compactly re-expressed as such,

$$H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})(1-1)} = \sum^{\text{all}}_{i_{(m_{|_{m\ge 1}^{\le M}})}} \sum^{\text{all}}_{\alpha_{i_{(m)}}} \sum_{\nu,\upsilon}^{\in\{\pm\}} \lambda_{i_{(m)}}^{\alpha_{i_{(m)}},\nu\upsilon} \mathcal{T}_{i_{(m)};\nu\upsilon}^{\alpha_{i_{(m)}}(1-1)} \tag{82}$$

with the so-defined *'Elementary Wheelerian Operators'*,

$$\mathcal{T}_{i_{(m)};\nu\upsilon}^{\alpha_{i_{(m)}}(1-1)} \equiv \breve{r}_{i_{(m)}}^{\alpha_{i_{(m)}},\nu} \breve{r}_{i_{(m)}}^{\dagger\alpha_{i_{(m)}},\upsilon} \tag{83}$$

As the indices manifest, the above operators $\mathcal{T}_{i_{(m)};\nu\upsilon}^{\alpha_{i_{(m)}}(1-1)}$ are defined for every subset of the participatory-observers $i_{(m)}$, and further, for every one $\alpha_{i_{(m)}}$ in the complete spectrum of their binary questions. By definition, for every such identification, the elementary Wheelerian operators either switch the two answering-chiralities, the off-diagonal ones, or simply witness-and-report the chiralities, the diagonal ones, in the present-moment global state of the purely-'it-from-qubit'-universe.

Now, utilizing (82,83), the total Hamiltonian of the simplest toy model of the Wheelerian Participatory Universe $H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})}$, takes the following form as a sub-evolution of (61),

$$H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})} = \sum_{s}^{1\le s\le M} H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})(s-s)}$$

$$H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})(s-s)} \equiv$$

$$\equiv \Big(\sum_{\text{all } i_{1(m_1)}}^{1\le m_1\le M} \sum_{\text{all } \alpha_{i_{1(m_1)}}} \sum_{\nu_1,\upsilon_1}^{\in\{\pm\}}\Big) \cdots \Big(\sum_{\text{all } i_{s(m_s)}}^{1\le m_s\le M} \sum_{\text{all } \alpha_{i_{s(m_s)}}} \sum_{\nu_s,\upsilon_s}^{\in\{\pm\}}\Big)$$

$$\lambda_{i_{1(m_1)}\cdots i_{s(m_s)}}^{\alpha_{i_{1(m_1)}}\cdots\alpha_{i_{s(m_s)}},\nu_1\upsilon_1\cdots\nu_s\upsilon_s} \mathcal{T}_{i_{1(m_1)};\nu_1\upsilon_1}^{\alpha_{i_{1(m_1)}}(1-1)} \cdots \mathcal{T}_{i_{s(m_s)};\nu_s\upsilon_s}^{\alpha_{i_{s(m_s)}}(1-1)} \tag{84}$$

Being one of the central characteristics of the unitary evolution (84), the yes-or-no answering-qubits $\breve{R}_{i_{(m\le M)}}^{\alpha_{i_{(m)}},\pm}$ do all interact with one another as conducted by the maximally-random many-body conversion operators of the theory $\mathcal{M}$.

Finally, we present the *'child sub-model' of the most minimalistic toy model of the HoloQuantum Networkical Wheelerian Participatory Universe*. As before, it is the one whose total Hamiltonian exponentiates (81),

$$H_M^{(\mathcal{W}.\mathcal{P}.\mathcal{U})\bigcirc} = \; = \exp\Big[\sum_{\text{all } i_{(m)}}^{1\le m\le M} \sum_{\text{all } \alpha_{i_{(m)}}} \sum_{\nu,\upsilon}^{\in\{\pm\}} \lambda_{i_{(m)}}^{\alpha_{i_{(m)}},\nu\upsilon} \mathcal{T}_{i_{(m)};\nu\upsilon}^{\alpha_{i_{(m)}}(1-1)}\Big] \tag{85}$$

VIII. HOLOQUANTUM NETWORK THEORY: THE GLOBAL OVERVIEW WITH CONNECTIONAL EXPLANATIONS

We begin this section with overviewing the perfected theory, and then make the connectional remarks.

HoloQuantum Network Theory, as we have defined and systematically formulated in this work, is the most fundamental and the most complete dynamical theory of all the entirely-quantized networks. From the purely physics point of view, $\mathcal{M}$ does the same job for physics which *the most complete 'it-from-qubit' theory* should do. To the best of our understanding, and as we propose here, HoloQuantum Network Theory is the most fundamental and most complete quantum many body theory by which 'all quantum natures', namely the entire quantum universe or multiverse, and also all her 'selective descendants' one by one, as obtained by all the 'phenomenological subsettings' together with all the 'observer-probes rescalings', are formulated information-theoretically in an all-unified and totally form-invariant way. All the quantum statics, all the microscopic interactions, and the total unitary quantum dynamics of the perfected $\mathcal{M}$ are directly sourced from the *unique nine principles* which, to the best of our undedrstanding, are both the unavoidable, 'the must be', and the most compelling, 'the best be', for its original intention to be entirely fulfilled.

Mathematically, the characters which define the theory are HoloQuantum Hypergraphs. These are the unitarily-evolving quantum states which are formed by superpositions of arbitrarily-structured hypergraphs whose vertices and hyperlinks are all quantum entities. The microscopic degrees of freedom of the theory are the purely information-theoretic qubits of the quantum vertices and the quantum hyperlinks, the vertons and the relatons, respectively.

As a whole, all the vertons and all the relatons form a pregeometric, all-fermionic, and *qualitatively novel* quantum many body system. The total dynamics of these vertons and relatons is sourced by a complete set of random many-body interactions which impartially occur in between all of them. Besides all the well-known interactions of 'conventional' multi-fermion-conversion operators, *HoloQuantum Network Theory has a whole lot of novel global microscopic interactions, being conducted by a hierarchical family of cascade operators*, implied by the evolving hypergraphic face of its total quantum many body system. Cascade operators, by which the hypergraphical weldefinedness of HoloQuantum Networks is dynamically safeguarded, also realize their quantum-hypergraphical isomorphisms.

Both mathematically and physically, all that $\mathcal{M}$ 'needs to' capture about the vertons and the m-relatons is that they are the defining qubits of the conceptually most-primitive states of 'absences and presences' of the maximally-flavored vertices and their hyperlinks. From this intrinsically-defining viewpoint, *the perfected theory is purely 'it-from-qubit', all the way from 'alpha to omega'.* Because of this central characteristic, and moreover because of the *covariant-completeness* of its total quantum statics and total unitary quantum dynamics, the perfected theory yields the most-complete formulation of how precisely the complete system of the yes-or-no qubits of the observer-participancy questions interact with one another and evolve, assembling the Entire Wheelerian Universe. The work presented here proposes $\mathcal{M}$, as the alternative to the approaches such as [5], *as the right theory to work-out 'in full concreteness' the all-explicit 'it-from-qubit' derivation of all physics.*

Serving as the conjectured theory of all physics, the quantized hypergraphs of HoloQuantum Network Theory represent the arbitrary choices of the quantum objects, by their vertons, and the arbitrary choices of the multi-object quantum relations, by their relatons. Purely relatonic operators $\mathcal{O}[\{\breve{r}_{(i^{a_i})_{(m)}}^{\alpha[(i^{a_i})_{(m)}]}, \breve{r}_{(i^{a_i})_{(m)}}^{\dagger\,\alpha[(i^{a_i})_{(m)}]}\}]$s, being correctly identified in every given context in terms of the fundamental relatons, can be the field operators of the arbitrarily-chosen physical interactions between the arbitrarily-chosen particles. Likewise, they can formulate all the statistical correlations, the functional relations, or the relational geometrical quantifications, of those particles. Most-generally, the complete set of the fundamental-or-emergent 'relational physical observables' one can measure for any arbitrarily-chosen particles, can be formulated by these operators. But, all the relational physical observables are definable only based on firstly defining some 'base-objects' for them. By including these objects into the dynamical quanta by the vertons, one arrives at the perfected theory. Being so, one formulates *all of physics as the HoloQuantum Networks of arbitrary objects-and-relations.*

Having made a review of HoloQuatum Network Theory, we now come to present, in the rest of this section, *a number of elucidating connectional remarks about some selected works in the literature* which has a highlighted overlap with some aspects of the perfected theory proposed here. In each case, we will comparatively comment and carefully elaborate not only on the notable conceptual or technical connections and similarities, but also on the multi-dimensional characteristic distinctions which HoloQuantum Network Theory makes with all those works. The works which we comparatively discuss in here belong to very different fields of quantum physics. This vast coverage, however, is very natural as the perfected theory is, by definition and construction, the form-invariant formulation of all quantum natures. We highlight that, regarding the selected works, these comparative remarks are *very subjective* and far from complete. This is so, firstly for not making this part overwhelmingly long, and secondly because the selected works are enough to make the productive points clear. These points shed light on how HoloQuantum Network Theory can advance all these fields.

Mathematically, the absolute primitivity, maximal generality, and intrinsic competence which distinguishes the defining framework of hypergraphs, suggest them to be the very primary characters of the 'pregeometry' [6] out of which the geometric space emerges. Graphs, namely hypergraphs with only the two-vertex-links, have a well-known history of being examined as models of pregeometry. Because the entire universe, and so also the fundamental 'setup' of the spacetime, is a quantum system, the hypergraphical pregeometry must be defined quantumly. Indeed, some interesting quantum models of graphical pregeometries have been already constructed in the past and recent literature [7–13]. These works, independent of the implemented quantum statistics of the degrees of freedom, *differently* but all *partially*, have employed some features of the total quantum many body system of HoloQuantum Networks. Both of the works [7, 8] already used the framework of second quantization for the graph-structural degrees of freedom, in [7] with some deterministic local evolution, and in [8] with some random interactions. The more recent and more advanced works of [11], looking for emergent locality and gravity, formulate initiative models of second-quantized quantum-graphical pregeometries, also including matter quanta, whose in-particular Hubbard-model-resembling structures-and-interactions develop interesting phases. The works of [12], using a second-quantized formalism being similar to the quantum-graphical methods of [11], present models of random complexes whose Markovian state-dependent unitary evolutions are realzied by gluing face-wise the simplices or the regular polytopes. Finally, the most recent works [13] offer simple toy models of the randomly-interacting graph-structural qubits whose ground-states, developed in the infrared quantum phase transitions, feature some four-diemsnional geometries.

Let us now elaborate collectively on *the significant points of distinctions* between what we have presented in this work and those presented in [7–13], as both the partial similarities between them and their partial overlaps must be already obvious to the reader.

First, HoloQuantum Network Theory is *a complete 'theory'.* By intention, this theory is constructed to serve as the fundamental and complete theory by which 'all of physics', and so also the complete theory of quantum pregeomtry and quantum gravity, can be formulated form-invariantly. Because it is neither a specific model, nor a collection of contextually-related models, its definition and formulation could not have been done in the arbitrary manners in which 'models' are built. Being so, nothing in this quantum theory is ad hoc, or has been implemented as a matter of examination, simplification, or specifications. Both the total statics and the complete unitary dynamics of the theory, as they must, are directly deduced from the merge of the stated nine principles required by the 'must-be'-and-'best-be'. This characteristic robust-and-complete determination is unlike all the ad-hoc structures implemented in the initiative models presnted in [7–13]. To highlight, all the many-body interactions of the theory are determined both uniquely and completely by the exact-or-almost symmetries, all the dynamical quantum constraints, the Wheelerian randomness, and finally the principle of the covariant-completeness. Indeed, *these very same microscopic interactions*, upon realizing all the extra structures or the extra symmetries which are needful to be imposed in concluding the complete theory of quantum gravity from within HoloQuantum Network Theory, *will also formulate form-invariantly the complete dynamics of the quantum spacetiem.* We will elaborate on this crucial point in section nine.

Second, we must remark here on a characteristic distinction regarding the inclusion of all the additional fundamental degrees of freedom which play the role of matter in quantum pregeometry and quantum gravity. On one hand, HoloQuantum Network Theory is 'purely internal'. It formulates a closed quantum many body system whose totality of the degrees of freedom, the vertons and relatons, are in a one-to-one map with the structures of the mathematically-quantized hypergraphs. On the other hand, by choosing arbitrarily a subsystem inside this very total quantum many body system, one formulates 'open HoloQuantum Networks', the chosen subsystem, which interact with their 'environments', their complementary subsystems. Such partitioned HoloQuantum Networks can naturally formulate 'the spacetime quanta' interacting with 'the matter quanta', in the quantum-gravitational implementtaions. Being so, *HoloQuantum Network Theory formulates both systems of 'pure quantum gravity' and 'interacting systems of quantum spacetime and quantum matter', in a manner which is microscopically and covariantly unified.*

Third, one independent dimension in which any HoloQuantum-Networkical formulation of the quantum pregeometry and quantum gravity must be *significantly distinct statically, by interactions and dynamically* from the models in [7–13], is the following. HoloQuantum Network Theory is, as it must be, a 'trans-graphical' theory, in fact, it is 'maximally hypergraphic'. This directly comes by the realization of principle seven, according to which *all the* $m^{>2}$*-relatons*, which by definition are 'non-reducible' to the two-relatons, are *as statically-fundamental and also as dynamically-active as the two-relatons in the theory.* Stated by principle seven, the theory must treat all the hypergraph-state-qubits with maximum-possoible equality. The minimalistic breaking of this symmetry is between the vertons and the relatons, only realized by the 'dressing' of relatons and by the cascade operators. Besides this, all qubits are equal. As such, by its total Hamiltonian, namely $H_M^{(\mathcal{FM})}$, all the $R_{i_{(m\ \in \mathbb{N}\leq M)}}$s are equally activated. As we propose, this '*all-relatons-equality*', being an exact fundamental symmetry of the theory, is instrumental for the correct theory of quantum gravity. Let us so conclude as follows. *The acceptable HoloQuantum-Networkical formulations of quantum pregeometry and quantum gravity are the ones which are 'maximally hypergraphic'.* Comparing $H_M^{(\mathcal{FM})}$ (61) with its counterparts in the aforementioned references proves the physical significance of this point.

Finally, let us conclude our connectional remarks on the models of quantum spacetime by the implication of *the covariant completeness* stated by principle nine. Because both the total state-pace (55) and the complete unitary dynamics (61) of the perfected theory do fully realize this principle, it is already guaranteed that all the network models whose statics and dynaimcs are 'quantumly correct' can be consistently embedded inside the totality of the perfected theory. This consistent embedding, of course, can be also checked concretely, example by example. In particular, *all the models presented in* [7–13] *are consistently embeddable inside* (55,61). To highlight, as explained in sections one to five and also in the review part of this section, this 'consistent-embedding criterion' holds independent of the quantum-statistical-identities of the objects or relations whose theory is information-theoretically formulated by the perfected theory. *That, all the 'independent' models in* [7–13] *are embeddable inside the Maximally-Flavored HoloQuantum Network Theory not only interconnects them fundamentally, but also makes all their consistent deformations and generalizations manifest.*

Now, we move on to the comparative remarks in regard with the SYK models [14–16], specially with their 'complexified versions' [15]. On one hand, an inclusive comparison will be made briefly. On the other hand, *this will highlight how one can generate the qualitatively highly-novel classes of quantum many body systems in the theory* $\mathcal{M}$.

Merely at the level of their formulations as quantum many body systems, HoloQuantum Networks do have both a number of notable *likenesses* and a number of significant characteristic *unlikenesses* with the extremely interesting SYK models, specially with their complexified formulations as given in [15]. We begin by highlighting *the formal likenesses*. The total Hilbert spaces are purely fermionic, in both cases. All the microscopic interactions, in both cases, are fundamentally 'all-to-all' and also random. Moreover, simply for the naturalness, but not restrictedly, the random couplings are all set to be Gaussianly distributed. Both HoloQuantum Network Theory and the 'complexified SYK' as formulated in [15] respect the fundamental symmetry of the global $U(1)$ transformations on the fundamental fermions. Finally, in both cases, the formulations are totally pregeometric. That is, the fermionic many body systems live in $(0+1)$ spacetime dimensions. We must emphasize again that these likenesses are at a *merely formal level*.

Let us now move on to highlight and elaborate briefly on *the characteristic unlikenesses* which, *even merely formulationally*, distinguish the works. We so begin with '*the least significant*' among these differences. Although in both cases the fundamental all-to-all interactions are all Gaussian-random, in HoloQuantum Network Theory, the independent couplings *'must be'* selected from the Gaussian-random distributions each one of which are to be fixed with tuning two arbitrary parameters, one for the mean values, and one for the standard deviations. Both in HoloQuantum Network Theory and in the model [15] together with the likewise-complexified formulation of the extended-SYK model in [16], the fundamental interactions are the many-fermionic-conversions. But, the total Hamiltonian of HoloQuantum Network Theory $H_M^{(\mathcal{FM})}$ concluded in (61) *'must necessarily' contain all* the $(m$ -to- $m)$ conversions impartially, to realize the covariant completeness stated by principle nine.

Now, we turn to remarking *the three unlikenesses* between the works which are '*the most significant*'. *First*, in the SYK models [14–16] all the fermionic degrees of freedom are treated equally, and this equal treatment which is both statical and dynamical, is held exactly. However, in HoloQuantum Network Theory this global 'equal-treatment symmetry' must be explicitly broken, as minimally as possible, but indeed unavoidably. The source of the explicit breaking of this symmetry is *a fundamental one*. In HoloQuantum Network Theory, the fermions F_I fundamentally belong to two categories, vertons and relatons. Relatons can be present or can be created only when their base-vertons are all present in the total quantum state of the network. Being so, all the relatons are 'system-state-conditional fermion qubits'. By principle seven, the equal-treatment symmetry is broken only and only by these relatonic conditionalities. *But, as* (61) *does manifest in comparison to* [14–16], *this effect strongly deforms the unitary dynamics* $H_M^{(\mathcal{FM})}$.

Second, because by seeing on its very mathematical face, the maximally-flavored HoloQuantum Network theory 'becomes equivalent with' the most complete theory of the all-structurally-quantized dynamical hypergraphs, the *quantum-hypergraphical isomorphisms* do play an important role in the construction of its dynamical side, as stated by principle five. Indeed, the realization of these complete-set isomorphism transformations has made a highly-significant impact on the definitions and the formulations of the microscopic interactions and the total unitary evolution of the perfected theory, as one can trace them back in the first five sections. But, in the SYK models [14–16] the relatonic structures are all trivial, so that this dynamically-impactful feature becomes effectively mute.

Third, the dynamical hypergraphical-welldefinedness of HoloQuantum Networks demands the hierarchical family of *cascade operators*. These highly-global operators play a central role in the fundamental interactions and so in the unitary evolution of the total many body system of the vertons and all the m-relatons. However, because the hypergraph structure of the SYK models is effectively trivial, *the interactions of HoloQuantum Network Theory and of the models* [14–16] *are majorly distinct at the microscopic level*, because of the both highly-frequent and highly-impactful presences of the cascade operators in $H_M^{(\mathcal{FM})}$ as concluded in (61).

Beyond all the above distinctions, HoloQuantum Network Theory is, by construction, the fundamental covariantly-complete theory in which every consistent quantum many body system is necessarily contained. *In regard with all the SYK-type models, in fact, these embeddings are very straightforward*. That is, not only the models [14–16] are immediately embeddable inside the totality of the Maximally-Flavored HoloQuantum Network Theory, but also all the descendant versions of them which are presented in the very recent literature, whether are generalized dimensionally, or are deformed statically or dynamically, can be easily embedded inside the very totality of (55,61). By embedding all these SYK-type models into HoloQuantum Network Theory, *all their consistent deformations and generalizations becomes manifest, interconnected and systematic*.

Finally, we highlight a point on the gravitational aspects of the SYK models [14, 15], connecting with our related points in section nine. The SYK models are already dual to some two-dimensional gravitational models. But, the perfectly-realized principle nine does guarantee that there must be a *sub-theory* of the Maximally-Flavored HoloQuantum Network Theory which holographically [18] defines and formulates the complete realistic theory of both classical and quantum gravity in the emergent realistic spacetimes in which, besides the single 'renormalization-group dimension', the other dimesnions are all developed *relatonically*.

IX. HOLOQUANTUM NETWORK THEORY: THE VISIONS OF TODAY AND FUTURE

HoloQuantum Network Theory, understood in *its 'minimum level'*, serves the whole quantum-granted physics the very same way that *category theory* serves the whole mathematics. But, it does serve physics *much more* than this, once being understood in *its 'maximum level'*. HoloQuantum Network Theory, by its definition, formulation and perfection, given the unique choice of its defining nine principles, serves as the right framework in which every *precedented-or-unprecedented* 'domain-specific' theory of physics can be firstly defined from the beginning and then be systematically built-up in a way which is *most 'direct', most 'fundamental' and most 'optimal'*. Being visited in the perspective of network science, HoloQuantum Network Theory is *'the' theory* which formulates every possible dynamical network, assuming that it respects all the laws of quantum physics structurally and functionally, which it surely does if being physically realizable.

In a way, it is by those unique nine principles which HoloQuantum Network Theory has been granted its intentional power, that is, the above *'minimum and maximum'*. The nine principles are characterized into two classes which are 'complementary'. *Six principles* among them, the *principles one, two, four, five, six and finally nine* are unavoidable for its minimal realization as *'the category theory of all physics'*. Being so, they belong to the 'must-be' class of these nine principles. In particular, we must highlight that, although the systematic construction of the perfected theory has been impacted, according to the principles five and six, by the realizations of the two distinct types of symmetry transformations, none of them is a 'beyond-categorical' feature. Clearly, to develop any quantum-hypergraphic categorical theory of arbitrary objects-and-relations, the most complete set of the quantum-hypergraphical isomorphisms must be realized by a (proper) subset of the Hamiltonian operators. Moreover, the $U(1)$ redundancies of the global phases must be demanded for any 'lowest-dimensional' categorical theory of quantum objects and their quantum relations to compute its observables correctly. Now, we come to the second class of the nine principles, namely the 'best-be'. They are the principles *seven, three and eight*. Principle seven, namely the principle of 'maximal hypergraphness' by which all the multi-degree quantum hyperlinks are equally-treated, and moreover all the quantum objects and the quantum relations are also treated as equally as it can be, is needed to make the perfected theory *'the optimal framework'* in which all the domain-specific theories of physics can be formulated. Finally, the Wheelerian principles three and eight are the ones which *uplift HoloQuantum Network Theory to its ultimate 'it-from-qubit' fulfillment* [1].

Summarized from this purely 'it-from-qubit' point of view, HoloQuantum Network Theory is the most general and the most complete dynamical interacting theory of the qubits for the absences-or-presences of 'absolutely whatever' of the objects and their relations in the entire quantum universe or the multiverse. Clearly, the most-complete time-dependent information of 'all quantum natures' is capturable by the total quantum many body system of a qualitatively sufficiently-diverse and quantitatively sufficiently-immense collection of the answring-qubits to the 'is-or-isn't' or equivalently, to the 'yes-or-no' questions. This is why, by its first-principle definition and perfected construction, $\mathcal{M}$ *is the most fundamental, the most general and the most complete Wheelerian theory of 'it-from-qubit'*. This, in particular, suggests that from within the prerfected theory, one can generate a whole families of novel more compelling sub-theories and models of quantum information and quantum computation. We will suffice to highlight three obvious directions in this fruitful territory. *Firstly*, one should be able to *reformulate and further genralize both statically and dynamically, the conventional quantum computation theory*, from within the totality of (55) and (61). *Secondly*, but relatedly, based on the results of [17] which are embeddable in, and generalizable by the total microscopic system of HoloQuantum Network Theory, one can devise *novel quantum-computation processors which can be practically superior*. *Thirdly*, upon a 'huge' completion of the simplest most-minimalistic toy-model of section seven, one can derive from within $\mathcal{M}$ *the complete theory of Wheelirian 'It-From-Qubit Observer-Participancy-Universe'*, an alternative to [5].

HoloQuantum Network Theory *must be* maximally minimalistic in taking its fundamental symmetries to safeguard its *maximal generality* as the fundamental quantum many body theory of *all* quantum natures. But, one can devise diverse models of HoloQuantum Networks endowed with the contextually-chosen extra symmetries. These contextual model buildings can be done in two ways. On one hand, as in the examples of section six, they can be obtained as the minimalizations being directly extracted from the total theory which has been fully perfected in section five. On the other hand, many forms of physically-significant extra symmetries, such as spacetime symmetries or the internal gauge symmetries, can also emerge as the qualitatively-distinct quantum-or-classical phases of HoloQuantum Networks in the enormous total phase diagram of the perfected theory. Also from a *purely network-science viewpoint*, in these very same two ways one can devise all models of unprecedented-or-precedented quantum-or-classical 'simple'-or-complex networks. By its clear significance for building all such sub-theories and models, to probe and progressively map *the distinct phases, fixed points, and phase transitions of the perfected HoloQuantum Networks* will be one of the most fruitful directions to explore in future.

As elaborated in section eight, being the most complete theory which formulates the most general time evolution of all the superpositions of the arbitrarily-structured quantized hypergraphs, HoloQuantum Network Theory is the most natural and the most fruitful framework to define, develop and conclude the ultimate formulation of *quantum pregeometry and the whole quantum gravity*. To accomplish this far-reaching goal from within the totality of (55,61), which conclude the perfected theory, is of course a whole grand project to be conducted, *surely requiring a number of totally-unprecedented ideas*. To advance in the correct direction toward this goal, we suffice to highlight here the one most-significant central characteristic which is already very well-appreciated. This distinctive feature is nothing but *'the correct form' of the most complete realization of Holography* [18], one must come up with. This intrinsics holographization is clearly one more remark on models of section eight. Let us highlight that the whole point in here is *the correct way* of realizing the strongest version of Holography, formulated in its purely-information-theoretic form and then implemented *as an 'extra' feature or structure on the whole generality of HoloQuantum Networks*. Namely, *'HoloQuantum-Networkian Theory of Quantum Gravity'* will be *a very specific 'sub-theory' of* the total HoloQuantum Network Theory which, by intrinsic construction, is immensely larger than a theory of only quantum gravity. This unmapped dimension must be explored in future works.

Being one *context-independent* direction of analysis within the framework of HoloQuantum Network Theory, one can systematically and precisely analyze *both the emergences and the characterizations of complexities* in a unique manner which is all-inclusive. By this we mean that, all the variants of complexities which are distinct phenomenologically, or are developed out of the different emergent mechanisms, can be unitedly formulated, classified and thoroughly understood in this unique framework. Given the extremely significant roles which *complex structures* play both in nature and in advanced technologies, this will be a distinctly-important field of study. In particular, because HoloQuantum Network Theory formulates a most general and most fundamental unification of (quantum and classical) many body systems and (quantum and classical) networks, one can now attempt to systematically interconnect the defining characteristics, the foundational principles, and the emergent mechanisms of complexities *in complex systems in physics and in complex networks*. Besides, by modeling *all* the computationally-distinct classes of the computation processors as some function-specific types of HoloQuantum Networks, one can reformulate unitedly and analyze more optimally *computational complexities in both mathematics and computer science*. Finally, HoloQuantum Network Theory, by being scale-covariant (on which we will elaborate next), is ideal for the study of the emergences of complexities even 'practically'.

HoloQuantum Network theory, by realizing all of its nine principles, is a quantum many body theory whose total statics, all microscopic interactions, and total unitary evolution do remain form-invariant in formulating all quantum natures. By the operationally-constructive definition of 'all quantum natures' given in section one, and also as explicitly stated in the explanatory note to the principle nine, this implies the following important statement. HoloQuantum Network Theory is completely form-invariant under changing the renormalization group scale all the way down from the ultraviolet fixed point. Therefore, the quantum equations of the perfected theory are all scale-'covariant'. Being so, one knows that *proposing HoloQuantum Network Theory as the most-fundamental most-complete covariant quantum many body theory of all quantum natures is not to be meant only in a 'scale-wise' conventional top-down manner*. Namely, although we can surely limit the perfected theory to function 'scale-by-scale', and so only in fixed scales, it can do much better in its functioning over the renormalization-group scales. Because of its realized covariant-completeness, *HoloQuantum Network Theory is intrinsically the 'Multi-Scale Theory' in which quantum objects-and-relations in the arbitrarily-chosen different renormalization-group scales can be all activated simultaneously and cooperate with one another*. Given the proven significance of the multi-scale-functioning complex systems both in nature and in the advanced technologies, it remain as an important mission for the future works *to manifestly formulate and thoroughly analyze, both at structural and functional levels, all the 'multi-scale organizational aspects' of the theory* $\mathcal{M}$.

HoloQuantum Network Theory, we must highlight, is being proposed in the present work as *the covariant 'quantum many body theory' of all the quantum natures at the most fundamental and the most complete level, but not as 'the ultimate theory'*. By this, one brings to attention that the whole quantum physics is taken for granted in HoloQuantum Network Theory, namely as its both fundamental and exact input. But, it may be that some defining aspects of quantum physics, or even all of them, are emergent form a still-unknown 'pre-quantum theory'. We suggest here that the problem of 'effectively undoing time' in HoloQuantum Network Theory, which we highlight in the conclusive paragraph of this paper, may serve as a good theoretical laboratory to experiment with the initiative models of the pre-quantum theory.

Finally, we must come to the very notion and the very role of *'time'* in the perfected theory $\mathcal{M}$. As a quantum theory, time plays the role of the 'external' evolution parameter in HoloQuantum Network Theory. But, we may be able to *relatonically 'internalize time'* into the microscopic body of HoloQuantum Networks. By this, we will be led to *a one-level-higher parental theory*.

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