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Effect of Static Disorder in an Electron Fabry–Perot Interferometer with Two Quantum Scattering Centers

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Abstract—In a recent paper—F. Ciccarello et al., New J. Phys. **8**, 214 (2006)—we have demonstrated that the electron transmission properties of a one-dimensional (1D) wire with two identical embedded spin–1/2 impurities can be significantly affected by entanglement between the spins of the scattering centers. Such an effect is of particular interest in the control of the transmission of quantum information in nanostructures and can be used as a detection scheme of maximally entangled states of two localized spins. In this letter, we relax the constraint that the two magnetic impurities are equal and investigate how the main results presented in the above paper are affected by a static disorder in the exchange coupling constants of the impurities. Good robustness against deviation from impurity symmetry is found for both the entanglement dependent transmission and the maximally entangled states generation scheme.

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The key role that entanglement plays in quantum information processing has been investigated over the past few years [1]. In this framework, the role that it plays in quantum transport in mesoscopic systems has been analyzed [2]. Recently, we have shown a novel way in which entanglement can be used for controlling electron transport in nanostructures [3]. Assume that we have a 1D wire, where two spin–1/2 impurities are embedded at a fixed distance. Such a system can be regarded as the electron analogue of a Fabry–Perot (FP) interferometer, with the impurities playing the role of two mirrors with a spin quantum degree of freedom. Single electrons are injected into the wire and undergo multiple scattering between the two magnetic impurities due to the presence of a contact exchange electron–impurity coupling. At each scattering event, spin-flip may occur and, thus, the transmitted spin state of the overall system will be generally different from the incoming one. The typical behavior shown by electron transmittivity T consists of a loss of electron coherence and, thus, of a resonance condition $T = 1$, due to the presence of internal spin degrees of freedom of the scattering centers [4]. Such a system is, indeed, the electron analogue of a Fabry–Perot (FP) interferometer, with the impurities playing the role of two mirrors with a spin quantum degree of freedom. However, unlike the standard FP device, where scattering with each mirror introduces a well-fixed phase shift, in the present system, the above phase shifts depend on the electron–impurities spin state and, thus, in general, a resonance condition cannot take place. However, the presence of quantum scatterers allows one to investigate if and to what extent maximally entangled states of the impurity spins can affect electron transmission. Denoting the triplet and singlet maximally entangled spin states of the impurities, respectively, by $|\Psi^{\pm}\rangle = 2^{-\frac{1}{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)$, we have, thus, found that when $|\Psi^{-}\rangle$ is prepared, a perfect resonance condition $T = 1$ can be always reached at electron wavevectors fulfilling $kx_0 = n\pi$ (where n is an integer and x_0 is the distance between the impurities) and regardless of the electron spin state. When this occurs, the incoming spin state of the electron–impurities system is transmitted completely unchanged. Therefore, a sort of perfect "transparency" takes place [3]. Moreover, as illustrated in Fig. 1, electron transmission within the one spin-up family of impurity states $\cos\vartheta|\uparrow\downarrow\rangle + e^{i\varphi}\sin\vartheta|\downarrow\uparrow\rangle$ is maximized (minimized) by $|\Psi^{-}\rangle(|\Psi^{+}\rangle)$. T is, thus, crucially affected by the relative phase φ. This suggests the appealing possibility to use entanglement between the impurity spins to control electron transmission in a 1D wire or, alternatively, to implement a maximally entangled states detection scheme via electron transmission. The above phenomena have been demonstrated to follow from an effective conservation law occurring whenever the con-

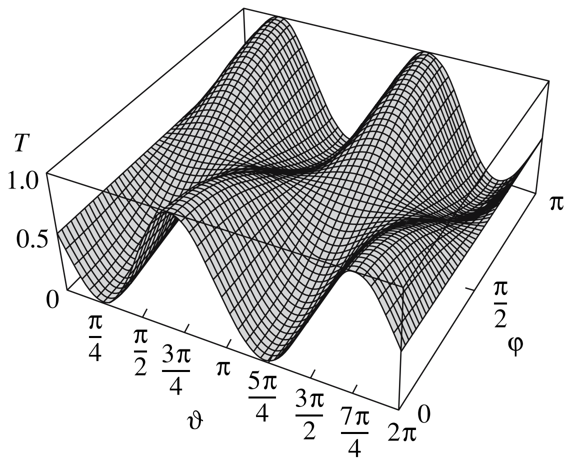

Fig. 1. Electron transmittivity T at $kx_0 = n\pi$ and $\rho(E)J = 10$ when the electron is injected in an arbitrary spin state with the impurities prepared in the state $\cos\vartheta|\uparrow\downarrow\rangle + e^{i\varphi}\sin\vartheta|\downarrow\uparrow\rangle$. $\rho(E)$ is the density of states per unit length of the wire.

dition $kx_0 = n\pi$ is fulfilled. In [3], the following Hamiltonian has been assumed:

$$H = \frac{p^2}{2m^*} + J\boldsymbol{\sigma}\cdot\mathbf{S}_1\delta(x) + J\boldsymbol{\sigma}\cdot\mathbf{S}_2\delta(x-x_0), \quad (1)$$

where $p = -i\hbar\nabla$, m^*, and $\boldsymbol{\sigma}$ are the electron momentum operator, effective mass, and spin–1/2 operator, respectively, $\mathbf{S}_i$ (i = 1, 2) is the spin–1/2 operator of the ith impurity, and J is the exchange spin–spin coupling constant between the electron and each impurity. Denoting the total spin of the system and of the two impurities, respectively, by $\mathbf{S} = \boldsymbol{\sigma} + \mathbf{S}_1 + \mathbf{S}_2$ and $\mathbf{S}_{12} = \mathbf{S}_1 + \mathbf{S}_2$, Hamiltonian (1) implies the conservation of $\mathbf{S}^2$ and S_z. $\mathbf{S}_{12}^2$ is, in general, not conserved due to the difference between $\delta(x)$ and $\delta(x - x_0)$ in (1). However, when $kx_0 = n\pi$, the effective representations $\delta_k(x)$ and $\delta_k(x - x_0)$ of these two electron orbital operators coincide (the electron being found at $x = 0$ and $x = x_0$ with equal probability) and $\mathbf{S}_{12}^2$ turns out to be an additional constant of motion. This fact is the ultimate reason for the occurrence of the above-mentioned behaviors associated with $|\Psi^-\rangle$ and $|\Psi^+\rangle$ (note that these are eigenstates of $\mathbf{S}_{12}^2$), as explained in detail in [3].

However, the above effective conservation law of $\mathbf{S}_{12}^2$ relies on the assumption of dealing with two perfectly identical impurities with equal coupling constant J. Of course, due to unavoidable static disorder, this condition cannot be strictly realized in a real system. The aim of this paper is to investigate how large the difference between the coupling constants of the two impurities can be before the entanglement dependent effects and the maximally entangled states generation scheme presented in [3] are significantly spoiled.

To begin with, let J_i (i = 1, 2) be the exchange coupling constant of the ith impurity. We, thus, generalize Hamiltonian (1) as

$$H = \frac{p^2}{2m^*} + J_1\boldsymbol{\sigma}\cdot\mathbf{S}_1\delta(x) + J_2\boldsymbol{\sigma}\cdot\mathbf{S}_2\delta(x-x_0). \quad (2)$$

It is convenient to introduce the quantities $\bar{J} = (J_1 + J_2)/2$ and $\Delta J = J_2 - J_1$ through which J_i (i = 1, 2) can be expressed as $J_1 = \bar{J} - \Delta J/2$ and $J_2 = \bar{J} + \Delta J/2$. Our previous results with identical impurities are, thus, recovered for $\Delta J \longrightarrow 0$. The exact stationary states of the system at all orders in J_1 and J_2 can be derived through an appropriate quantum waveguide theory approach. Since $\mathbf{S}^2$ and S_z are still constants of motion when $J_1 \neq J_2$, the block diagonalization-based procedure used for the case of two identical impurities [3] can be readopted, the difference being that, in the present case, there is an additional parameter. Denoting the total spin of the electron and the ith impurity as $\mathbf{S}_{ei} = \boldsymbol{\sigma} + \mathbf{S}_i$ and assuming left-incident electrons, we use as the spin–space basis the states $|s_{e2}; s, m_s\rangle$, common eigenstates of $\mathbf{S}_{e2}^2$, $\mathbf{S}^2$, and S_z, to express, for a fixed wavevector $k > 0$, each of the eight stationary states of the system as an 8D column. The calculation of the stationary states through suitable boundary conditions [3] allows us to find all of the transmission probability amplitudes $t_{s_{e2}}^{(s'_{e2},\, s)}$ that an electron prepared in the incoming state $|k\rangle|s'_{e2}; s, m_s\rangle$ is transmitted in the state $|k\rangle|s_{e2}; s, m_s\rangle$. These coefficients can be used to compute how an electron is transmitted through the wire for any arbitrary initial spin state of the system [3]. The transmission amplitudes $t_{s_{e2}}^{(s'_{e2},\, s)}$ turn out to be functions of the three dimensionless parameters kx_0, $\rho(E)\bar{J}$, and $\rho(E)\Delta J$, where $\rho(E) = (\sqrt{2m^*/E})/\pi\hbar$ is the density of states per unit length of the wire as a function of electron energy E.

We begin our analysis by investigating how the effect of perfect transparency shown when the impurity spins are prepared in the singlet state is affected by a difference in the two coupling constants. In Fig. 2, we plot the electron transmittivity T versus $\rho(E)\bar{J}$ and $\rho(E)\Delta J$ when the electron is injected in an arbitrary spin state for $kx_0 = n\pi$ with the impurities in the state $|\Psi^-\rangle$. Note that $T = 1$ for $\Delta J = 0$, since the case of perfect transparency with identical impurities is recovered. As expected, for a fixed $\rho(E)\bar{J}$, T decreases for increasing values of $|\Delta J|$ due to the progressive lack of conservation of $\mathbf{S}_{12}^2$. Note how this decrease gets faster for increasing strengths of $\rho(E)\bar{J}$, indicating that, for a

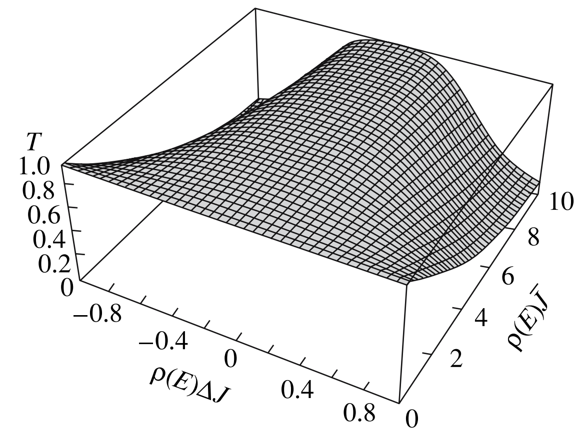

Fig. 2. T versus $\rho(E)\bar{J}$ and $\rho(E)\Delta J$ for $kx_0 = n\pi$ when the electron is injected in an arbitrary spin state with the impurities prepared in the state $|\Psi^-\rangle$. $\rho(E)\Delta J$ is normalized to $\rho(E)\bar{J}$, reducing to the ratio $\Delta J/\bar{J}$.

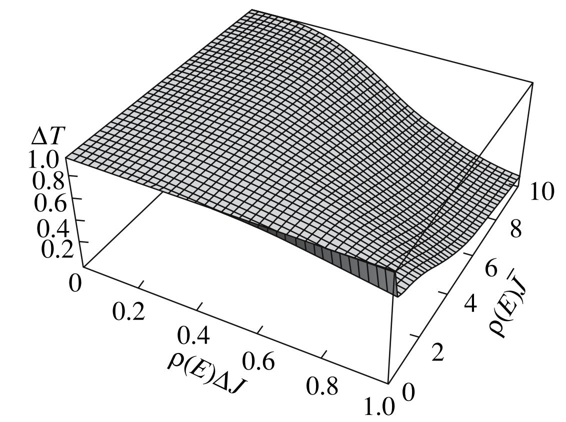

Fig. 3. $\Delta T = T_{\Psi^-} - T_{\Psi^+}$ versus $\rho(E)\bar{J}$ and $\rho(E)\Delta J$ for $kx_0 = n\pi$ when the electron is injected in an arbitrary spin state. $\rho(E)\Delta J$ is normalized to $\rho(E)\bar{J}$, reducing to the ratio $\Delta J/\bar{J}$. ΔT is normalized to its value for $\Delta J = 0$.

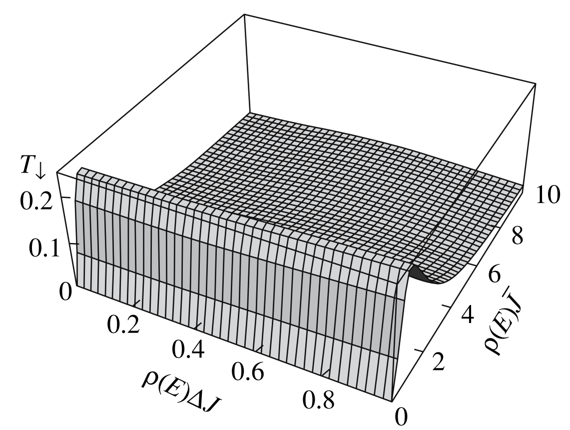

Fig. 4. $T_{\downarrow}$ versus $\rho(E)\bar{J}$ and $\rho(E)\Delta J$ for $kx_0 = n\pi$ when the electron is injected in the state $|\uparrow\rangle$ with the impurities prepared in the state $|\downarrow\downarrow\rangle$. $\rho(E)\Delta J$ is normalized to $\rho(E)\bar{J}$, reducing to the ratio $\Delta J/\bar{J}$.

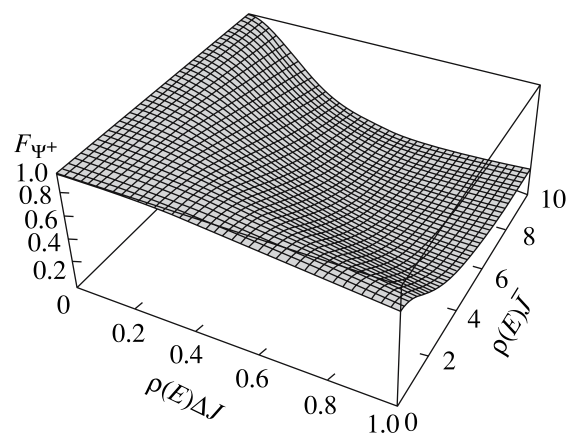

Fig. 5. F_{Ψ^+} versus $\rho(E)\bar{J}$ and $\rho(E)\Delta J$ for $kx_0 = n\pi$ when the electron is injected in the state $|\uparrow\rangle$ with the impurities prepared in the state $|\downarrow\downarrow\rangle$. $\rho(E)\Delta J$ is normalized to $\rho(E)\bar{J}$, reducing to the ratio $\Delta J/\bar{J}$.

given electron energy, low coupling constants $\bar{J}$ show better robustness against impurities asymmetry. It turns out that, for a difference in the coupling constants larger than 25% compared to $\bar{J}$, perfect transparency is not significantly spoiled ($T > 0.95$) in the whole broad range of strengths of $\rho(E)\bar{J}$ considered here.

A remarkable feature of the plot in Fig. 2 is its symmetry with respect to a change of sign in ΔJ for a fixed $\bar{J}$ (the relevant parameter being, thus, $|\Delta J|$). To explain this, we recall that, for $kx_0 = n\pi$, $\delta_k(x) = \delta_k(x - x_0)$, and write the nonkinetic part V of Hamiltonian (2) in the form

$$V = \left[\bar{J}\boldsymbol{\sigma}\cdot(\mathbf{S}_1 + \mathbf{S}_2) + \frac{\Delta J}{2}\boldsymbol{\sigma}\cdot(\mathbf{S}_2 - \mathbf{S}_1)\right]\delta_k(x). \quad (3)$$

Note that, in such regime, where the electron has an equal probability of being found at $x = 0$ and at $x = x_0$, a change in the sign of ΔJ is equivalent to an interchange of the impurity indexes. Therefore, the above symmetry property of T in the case of Fig. 2 straightforwardly follows from the symmetry of $|\Psi^-\rangle$ under an interchange of impurity 1 and 2. In the remainder of this paper, we will, thus, consider only positive values of ΔJ

whenever the initial spin state is symmetric for an interchange of two impurities, such as in the cases of $|\Psi^+\rangle$ and $|\downarrow\downarrow\rangle$.

Denoting the value of T obtained for $|\Psi^\pm\rangle$ by $T_{\Psi^\pm}$, it is worth analyzing the behavior of $\Delta T = T_{\Psi^-} - T_{\Psi^+}$, that is, the difference of the electron transmittivity for $|\Psi^-\rangle$ (high transmission) and $|\Psi^+\rangle$ (low transmission). In order to observe the entanglement-controlled transmittivity and/or detect the maximally entangled singlet/triplet states, one aims at having the highest possible value of ΔT [3] with the hope that it is only weakly affected by some impurities' asymmetry. Regarding the latter issue, in Fig. 3, we plot ΔT normalized to its value for $\Delta J = 0$ versus $\rho(E)\bar{J}$ and $\rho(E)\Delta J$ in the same regime considered in Fig. 2. A behavior qualitatively very similar to the case of Fig. 2 is exhibited. In the whole considered range of $\rho(E)\bar{J}$, ΔT turns out to be only weakly affected ($\Delta T > 0.95$) by a difference in the impurity coupling constants up to more than 25%.

To observe the entanglement-dependent electron transmittivity, one must, of course, be able to prepare the maximally entangled states $|\Psi^-\rangle$ and $|\Psi^+\rangle$. These states can be easily transformed into each other by simply introducing a relative phase shift through a local field. In [3], we have proposed a scheme to generate the state $|\Psi^+\rangle$ via electron scattering, improving a previous recent proposal [5]. The idea is to inject an electron in the state $|\uparrow\rangle$ in the regime $kx_0 = n\pi$ with the two impurity spins initially in state $|\downarrow\downarrow\rangle$. Due to conservation of both $\mathbf{S}_{12}^2$ and S_z, when the electron is transmitted in state $|\downarrow\rangle$ with probability $T_\downarrow$, the two impurities are projected into the state $|\Psi^+\rangle$ [3]. A difference in the coupling constants of the impurities is expected to modify the spin-polarized transmission probability $T_\downarrow$. Moreover, since the scheme relies on the conservation of $\mathbf{S}_{12}^2$, the two localized spins, in general, will not be projected in the maximally entangled state $|\Psi^+\rangle$. In Figs. 4 and 5, we, thus, plot, $T_\downarrow$ and the fidelity F_{Ψ^+} with respect to $|\Psi^+\rangle$ of the (normalized) spin state, respectively, into which the impurities are projected after the electron is transmitted in the spin down state. $T_\downarrow$ is almost negligibly affected by the presence of ΔJ. The same is not true for F_{Ψ^+}, which is, indeed, expected to be very sensible to the lack of conservation of $\mathbf{S}_{12}^2$. $F_{\Psi^+} > 0.95$ in the whole considered range of $\rho(E)\bar{J}$ for a difference in the impurity coupling constants up to more than 5%. However, for $\rho(E)\bar{J} \simeq 1$, that is, the strength of the impurity coupling constant maximizing $T_\downarrow$, $F_{\Psi^+} > 0.95$ up to values of $|\Delta J|/\bar{J}$ larger than 30%.

In conclusion, the results presented in this paper show very good tolerance of the entanglement-dependent transmission effects occurring in a 1D wire with two spin–1/2 impurities [3] with respect to unavoidable static disorder in the coupling constants of the impurities. Therefore, the experimental difficulty in realizing two perfectly identical magnetic impurities does not appear to be an obstacle for the observation of such phenomena.

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