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Cosmology of F-theory GUTs

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Abstract: In this paper we study the interplay between the recently proposed F-theory GUTs and cosmology. Despite the fact that the parameter range for F-theory GUT models is very narrow, we find that F-theory GUTs beautifully satisfy most cosmological constraints without any further restrictions. The viability of the scenario hinges on the interplay between various components of the axion supermultiplet, which in F-theory GUTs is also responsible for breaking supersymmetry. In these models, the gravitino is the LSP and develops a mass by eating the axino mode. The radial component of the axion supermultiplet known as the saxion typically begins to oscillate in the early Universe, eventually coming to dominate the energy density. Its decay reheats the Universe to a temperature of $\sim 1\,\text{GeV}$, igniting BBN and diluting all thermal relics such as the gravitino by a factor of $\sim 10^{-4} - 10^{-5}$ such that gravitinos contribute a sizable component of the dark matter. In certain cases, non-thermally produced relics such as the axion, or gravitinos generated from the decay of the saxion can also contribute to the abundance of dark matter. Remarkably enough, this cosmological scenario turns out to be independent of the initial reheating temperature of the Universe. This is due to the fact that the initial oscillation temperature of the saxion coincides with the freeze out temperature for gravitinos in F-theory GUTs. We also find that saxion dilution is compatible with generating the desired baryon asymmetry from standard leptogenesis. Finally, the gravitino mass range in F-theory GUTs is $10 - 100\,\text{MeV}$, which interestingly coincides with the window of values required for the decay of the NLSP to solve the problem of ^7Li over-production.

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JHEP04(2010)054

Contents

1 Introduction

Two prominent triumphs of modern theoretical physics are the Standard Models of particle physics and cosmology. Moreover, the interplay between particle physics and astrophysics/early Universe cosmology has already proven to be a fruitful arena of investigation for both fields. On the astrophysics side, this interplay has led to the very successful predictions of Big Bang Nucleosynthesis (BBN), which accounts for the abundance of light elements. In addition, ideas from particle physics have provided several plausible mechanisms such as baryogenesis or leptogenesis which could generate the observed baryon asymmetry. Finally, many extensions of the Standard Model include dark matter candidates.

On the particle physics side, constraints from astrophysics have led to novel, and sometimes quite stringent conditions on possible extensions of the Standard Model. These constraints can translate into important bounds on parameters of a candidate model which may be inaccessible from other avenues of investigation, and which can have repercussions beyond their immediate astrophysics applications. For example, compatibility with the successful predictions of BBN imposes important restrictions such as the requirement — spectacularly confirmed by LEP — that essentially there are at most three generations of light neutrinos! This is in amazing accord with the Standard Model of particle physics, and indicates a deep link between these two fields. Other cosmological considerations such as over-production of gravitinos in supersymmetric models, or a deficit in the observed baryon asymmetry provide additional constraints. Satisfying all of these constraints is often a non-trivial task for a given model, but can also point the way to novel mechanisms which may not be available in the standard cosmology.

At a more refined level, the interrelations between particle physics and cosmology roughly bifurcate into issues where gravity itself plays a central role, and questions where gravity plays only a supporting role in addressing more detailed features of a given particle physics model. For example, issues connected to the cosmological constant, or the homogeneity of the early Universe fall in the former category, whereas particle physics oriented issues such as the origin of dark matter or the overall baryon asymmetry fit most naturally in the latter category. Due to the vast array of observational data from probes wholly separate from cosmology, issues more closely tied to particle physics appear to at present be more tractable. In this regard, it is therefore quite natural, as is often done in the particle physics literature, to exclusively focus on the "low energy" aspects associated with the cosmology of a given particle physics model, parameterizing our ignorance of what occurs in the very early Universe in terms of an "initial reheating temperature" for the Universe, $T^0_{\rm RH}$. For the purposes of particle physics considerations, the cosmology of the Universe effectively begins at this temperature. In many cases, this reheating temperature ends up being smaller than the Planck, or GUT scale, and models with $T^0_{\rm RH}$ as low as $10-100\,\mathrm{TeV}$ have also been discussed.

It is interesting that this division in cosmology between gravity and particle physics issues parallels recent developments in the string theory literature. It is clear that a vast landscape of consistent string theory vacua exists. While this makes the string theory paradigm a very rich and flexible physical model, at present it also lacks predictive power because it does not lead to any particularly distinguished vacua! As a principle which can be used to limit the search for semi-realistic vacua, in [1–5] (see also [6–17]), it was shown that in the context of F-theory based models, demanding the existence of a limit where the Planck scale (and thus associated gravitational questions) decouples, in tandem with some qualitative particle physics considerations such as the existence of a GUT, leads to a remarkably limited, and predictive framework. In fact, without adding any additional ingredients, traditionally vexing particle physics issues related to flavor hierarchies, the doublet-triplet splitting problem, the $\mu/B\mu$-problem, and undesirable GUT mass relations all find natural solutions in F-theory GUT models. In addition, natural estimates for the overall magnitude of the Yukawa couplings, axion decay constant, μ parameter and MSSM soft mass terms all fall in an acceptable phenomenological range.

The aim of the present paper is to examine the cosmology of the F-theory GUT scenario as a model of particle physics. We find that these models naturally satisfy the typically stringent constraints derived from compatibility with BBN as well as bounds on the overall relic abundance of dark matter candidates, such as the gravitino. Perhaps the most striking feature of this analysis is the *absence* of any major problem, namely, that *consistency with cosmology imposes almost no constraint at all on the initial reheating temperature* T^0_{RH}.

That this is the case is a highly non-trivial consequence of the parameter range found for F-theory GUTs. Indeed, one potentially significant source of tension could in principle have originated from the fact that as a model of gauge mediated supersymmetry breaking with a relatively high mass for the gravitino [3]:

$$m_{3/2} = \frac{1}{\sqrt{3}}\frac{F}{M_{\mathrm{PL}}} \sim 10 - 100 \text{ MeV}, \tag{1.1}$$

the relic abundance of gravitinos, which is the LSP, can potentially overclose the Universe. In the above, as throughout this paper, M_{PL} denotes the reduced Planck mass $M_{\mathrm{PL}} = 2.4 \times 10^{18}$ GeV, and F denotes the component of the GUT group singlet chiral superfield X responsible for supersymmetry breaking:

$$\langle X \rangle = x + \theta^2 F \tag{1.2}$$

where as shown in [3], simultaneously solving the μ problem and generating viable soft mass terms in a minimal gauge mediation scenario requires:

$$F \sim 10^{17} \text{ GeV}^2 \tag{1.3}$$

$$x \sim 10^{12} \text{ GeV}. \tag{1.4}$$

We note that the Goldstino mode corresponding to the fermionic component of X is eaten by the gravitino.

In many gauge mediation models, the scale of supersymmetry breaking is significantly lower. From the perspective of cosmology, the relic abundance of gravitinos would at first appear to provide strong motivation for lowering the value of $m_{3/2}$. Indeed, in the most straightforward approximation, it is quite natural to take T^0_{RH} as high as the GUT scale. In this case, a well known estimate for the relic abundance of gravitinos in many supersymmetric models requires:

$$\frac{m_{3/2}}{2\ \text{keV}} \leq 0.1 \tag{1.5}$$

to avoid an overabundance of gravitinos in the present Universe. In particular, this would appear to suggest an upper bound for F of order:

$$F \lesssim 7 \times 10^{11}\ \text{GeV}^2 \tag{1.6}$$

which is significantly lower than equation (1.3)! In the gravitino cosmology literature, it is common to take $T^0_{\text{RH}} \ll M_{\text{GUT}} \sim 3 \times 10^{16}\,\text{GeV}$ to truncate the production of thermally produced gravitinos, thus avoiding precisely these types of issues. At a conceptual level, though, it is somewhat distressing that particle physics considerations demand such a stringent upper bound on T^0_{RH}. Indeed, insofar as the value of T^0_{RH} is dictated by gravitational issues which are a priori wholly separate from details of a particular particle physics models, this type of tuning of parameters is quite puzzling.

In F-theory GUTs, the resolution of this apparent dilemma again resides in the chiral superfield X. The essential point is that X plays a dual role in F-theory GUTs because its bosonic component breaks the anomalous global U(1) Peccei-Quinn symmetry of the low energy theory. As such, the associated Goldstone mode is the QCD axion, with decay constant:

$$f_a \sim \sqrt{2}x \sim 10^{12}\ \text{GeV}, \tag{1.7}$$

solving the strong CP problem. In the context of supersymmetric theories, however, the axion corresponds to one of two real degrees of freedom associated with the bosonic component of the corresponding axion supermultiplet. The other degree of freedom, known as the saxion is exactly massless in the limit where supersymmetry is restored, and in the present context has a mass of order 100 GeV. An exciting feature of the F-theory GUT is that the mass of the saxion is controlled by UV sensitive details of the compactification, such as the mass of the anomalous $\text{U}(1)_{\text{PQ}}$ gauge boson. As such, cosmological constraints for the saxion provide a window into the high scale dynamics of the model.

Because the potential of the saxion is nearly flat, it is easily displaced from its minimum, and will generically begin to oscillate as the early Universe cools from the initial reheating temperature T^0_{RH} until the saxion decays. In a generic situation, the initial amplitude for s_0 is sufficiently large that its vacuum energy density comes to dominate the energy density of the Universe. The value of s_0 is on the order of the characteristic Kaluza-Klein scale for the X field:

$$s_0 \sim M_X \sim 10^{15.5}\ \text{GeV}. \tag{1.8}$$

The decay of the saxion releases a large amount of entropy into the Universe, effectively diluting the overall relic abundance of all particle species. As a consequence, we find that rather neatly, one component of the axion supermultiplet, the saxion, counteracts the potentially dangerous features of the fermionic gravitino component (which includes the axino as the longitudinal degree of freedom)! We note that the decay of the saxion or some other cosmological modulus as a means to dilute the relic abundance of a particle species to acceptable levels has certainly been discussed in the literature before, for example in [18]. The primary novelty here is that without any additional assumptions, *F-theory GUT models automatically resolve the most problematic features of gravitino cosmology.* The requisite era of saxion dominance occurs provided the initial reheating temperature satisfies:

$$T^0_{\rm RH} \gtrsim 10^6 \text{ GeV}. \tag{1.9}$$

In this case, the dilution of the saxion is such that the gravitino could naturally make up a prominent component of the dark matter density. Depending on whether the axion begins to oscillate before or after an era of saxion domination, the axion can also provide a component of the dark matter. We are currently investigating whether F-theory GUTs provide additional dark matter candidates [19]. Finally, we also find that lower values of the reheating temperature are also possible, and are compatible with a regime where the saxion has a smaller initial amplitude. In this case, the gravitino and axion can potentially both contribute to the dark matter density.

But while the evolution of the saxion neatly solves the gravitino "problem", it can in principle introduce additional constraints. For example, the decay of the saxion, with its significant entropy release might disrupt the start of BBN. Most conservatively, this requires that the reheating temperature for the saxion remain above the starting temperature for BBN. Quite remarkably, this requirement is again satisfied quite naturally in F-theory GUT models. We find that the reheating temperature of the saxion, $T^s_{\rm RH}$ satisfies:

$$T^s_{\rm RH} \sim 1 \text{ GeV} > T_{\rm BBN} \sim 2 \text{ MeV}. \tag{1.10}$$

Compatibility with BBN also imposes strong constraints on the decay products of the saxion. Using the well known bound (which we shall review) on the branching ratio of the saxion to relativistic species such as the axion, we find that we must either posit the existence of an additional decay product not found in the MSSM, or that the saxion must have sufficient mass to decay dominantly to two Higgs fields so that:

$$m_{\rm sax} > 2m_{h^0} \sim 230 \text{ GeV}. \tag{1.11}$$

A sufficient amount of baryon asymmetry must be generated at high temperatures in order for BBN to produce the observed abundances of light elements. In the standard solution to the gravitino problem, lowering the initial reheating temperature $T^0_{\rm RH}$ has the deleterious consequence of removing some of the more efficient mechanisms for generating such an asymmetry, such as GUT scale baryogenesis, or standard leptogenesis. Rather, in this context it is common to invoke a mechanism where a large baryon asymmetry can be generated by the coherent oscillation of a field which carries either non-trivial baryon, or lepton number, such as in the Affleck-Dine scenario. In the present context, however, the dilution of the saxion completely relaxes any upper bound on $T^0_{\rm RH}$. Thus, the only condition left to check is whether any of the available mechanisms are capable of generating a sufficiently large initial baryon asymmetry which can survive the effects of dilution. Performing this analysis, we find that standard leptogenesis indeed produces an appropriate baryon asymmetry. In other words, the range of parameters suggested by

JHEP04(2010)054

F-theory GUTs naturally fall within the small range of dilution factors compatible with diluting the gravitinos to an amount where they do not overclose the Universe, but which nevertheless preserve a sufficient baryon asymmetry.[1]

The organization of this paper is as follows. In section 2 we review the main interrelations between cosmology and particle physics which will occupy a central role in the present paper. In section 3 we study the axion supermultiplet and its interactions in the context of F-theory GUTs. Section 4 forms the main body of our paper in which we present the particular cosmological scenario suggested by F-theory GUTs, paying special attention to the role of the saxion. In this same section, we analyze the gravitino and axion relic abundance, study constraints from BBN, and determine the overall baryon asymmetry generated by standard leptogenesis. Finally, in section 5 we briefly discuss future avenues of investigation in the cosmology of F-theory GUTs.

2 Cosmology and particle physics

One of the primary aims of the present paper is to determine how cosmological considerations constrain F-theory GUTs. In this section we review the main issues which must be addressed in a viable cosmological scenario. This section is entirely review, and can safely be skipped by the reader familiar with this material.

We first describe the main concepts which will figure prominently in the estimates to follow. After this, we proceed to a more detailed review of the features which will be especially important for analyzing the cosmology of F-theory GUTs. To this end, in the next subsection we provide a brief introduction to the FRW Universe, next describing the history of the standard cosmology as the Universe evolves in such a Universe. As will be apparent in later sections, the cosmology of the gravitino plays an especially important role in assessing the viability of a supersymmetric model. For this reason, we next review the computation of relic abundances, and in particular, describe in some detail the usual "gravitino problem". While this is indeed a significant constraint on many models, in certain situations the dilution due to a late decaying cosmological modulus can significantly alter this analysis. With this in mind, we next review the primary consequences of late-decaying cosmological moduli. We next review the constraint on the number of thermalized relativistic species derived from BBN. Finally, we conclude this section with a discussion of more refined features of BBN which are present in models such as F-theory GUTs which possess a late-decaying NLSP.

2.1 FRW universe

Below the scale of compactification, and before the present epoch, to good approximation, the early Universe is described by the four-dimensional Robertson-Walker (RW) metric:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right), \tag{2.1}$$

[1]See [20] for other recent work on moduli stabilization, leptogenesis, and dark matter in the context of gauge and gravity mediated scenarios.

where in the above, we have introduced the scale factor $a(t)$, as well as the curvature constant k, which after a suitable rescaling takes the values $k = +1, 0, -1$ for a respectively closed, flat, or open Universe. This describes the evolution of an isotropic Universe with homogeneous energy distribution. The overall expansion rate of the Universe is measured by the Hubble parameter:

$$H \equiv \frac{\dot{a}}{a}. \tag{2.2}$$

It is also common to introduce a dimensionless variant of H, called h defined by the equation:

$$H = 100h \text{ km Mpc}^{-1} \text{ sec}^{-1}. \tag{2.3}$$

The present value of h is given by [21]:

$$h_0 \sim 0.7. \tag{2.4}$$

When the context is clear, we will often drop this subscript to avoid cluttering various equations.

The background energy density determines the expansion rate of the Universe via the Friedmann equations:

$$\ddot{a} = -\frac{4\pi}{3} G_N(\rho + 3p)a \tag{2.5}$$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} \tag{2.6}$$

where in the above, p denotes the pressure of the given "fluid", G_N denotes the four-dimensional Newton's constant, and ρ corresponds to the total energy density of the Universe. The critical density ρ_c is defined as the value of the total energy density for which $k = 0$, so that:

$$\rho_c = \frac{3H^2}{8\pi G_N}. \tag{2.7}$$

Note that the critical density is a non-trivial function of t.

The total energy density will in general receive various types of contributions, so that ρ is given by a sum of the form:

$$\rho = \sum_i \rho_i. \tag{2.8}$$

Plugging this expression into equation (2.6) now yields:

$$\Omega_{\text{tot}} = \sum_i \Omega_i = 1 + \frac{k}{H^2 a^2} = 1 + \frac{k}{\dot{a}^2} \tag{2.9}$$

where we have introduced the parameter:

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}. \tag{2.10}$$

The sign of k correlates with the magnitude of Ω_{tot}:

$$\Omega_{\text{tot}} > 1 : k = +1 \tag{2.11}$$

$$\Omega_{\text{tot}} = 1 : k = 0 \tag{2.12}$$

$$\Omega_{\text{tot}} < 1 : k = -1. \tag{2.13}$$

JHEP04(2010)054

Due to the overall dependence on H^2 in ρ_c, it is sometimes convenient to introduce the quantity $\Omega_i h^2$, with $h = \kappa H$ with κ the constant implicitly defined by equation (2.3). The overall dependence on H^2 factors out, yielding:

$$\Omega_i h^2 \equiv \frac{\rho_i}{\rho_c} \cdot (\kappa H)^2 = \frac{8\pi G_N \kappa^2}{3} \cdot \rho_i. \tag{2.14}$$

The energy density of the i^{th} species also evolves with scale. In the approximation where the energy density ρ_i is proportional to the pressure p_i, the equation of state is given by:

$$p_i = w_i \rho_i, \tag{2.15}$$

for some constant w_i. The scaling behavior of ρ for three common choices is:

$$\rho_r \propto a^{-4} : w_r = 1/3 \tag{2.16}$$

$$\rho_m \propto a^{-3} : w_m = 0 \tag{2.17}$$

$$\rho_\Lambda \propto \text{const} : w_\Lambda = -1 \tag{2.18}$$

where r, m and Λ respectively denote "radiation" or relativistic matter, non-relativistic matter, and vacuum energy density.

Observation indicates that the Universe has recently transitioned from an era of matter domination to one where a background vacuum energy density plays a dominant role, with [21]:

$$\Omega_\Lambda \sim 0.7 \tag{2.19}$$

$$\Omega_m \sim 0.3. \tag{2.20}$$

The matter content Ω_m further subdivides into a subdominant "visible" matter contribution with $\Omega_{\text{visible}} \sim 0.05$) with the rest being comprised of the so-called dark matter $\Omega_{\text{DM}} \sim 0.25$, which by definition interacts weakly with the Standard Model degrees of freedom. The existence of such a large additional component of matter indicates that the Standard Model must be extended in some fashion. An important feature of this is that the overall energy density is quite close to the critical value:

$$\Omega_{\text{tot}} = \Omega_\Lambda + \Omega_m \sim 1. \tag{2.21}$$

2.2 Timeline of the standard cosmology

Having reviewed the main concepts which we shall use throughout this paper, we now describe in reverse chronological order the timeline for the standard cosmology. At various points we also indicate where deviations from this trajectory are possible. This material can be found in many standard textbooks, such as [22, 23], for example.

Era of structure formation: $T \sim 1$ meV$\rightarrow$ 100 meV. At present, the Universe is roughly 10^{18} sec old and the background photon radiation is at a temperature of 2.7 K ~ 1 meV, which is presently characterized as an era where large scale structures have formed. In addition, the energy density of the Universe is composed of roughly 5% visible baryonic matter, 25% non-baryonic matter, or "dark matter", and 70% of some other type of energy density which appears to share the characteristics of a vacuum energy density, or cosmological constant. At this point, the abundances of all of the light elements have been produced via cosmological processes earlier in the history of the universe, as well as other, more astrophysical processes having to do with the birth and death of stars, for example. At somewhat earlier timescales around 10^{16} sec described by a temperature of a few hundred meV, the present galaxies begin to form.

Era of BBN and atom formation: $T \sim 1\,\mathrm{eV} \to 5$ MeV. Prior to the start of structure formation, radiation and matter decouple at roughly 10^{12} sec and a temperature of $\sim 1\,\mathrm{eV}$. At this point, all of the present atoms have formed and will soon begin to form large scale structure. At only slightly earlier times, or higher temperatures, the universe is sufficiently hot to overcome the binding energy of atoms. At this point, nucleosynthesis has finished, and an appropriate abundance of the ions such as H^+, D^+, T^+, $^3He^{++}$, $^4He^{++}$, $^7Li^{+++}$ have formed.[2] These ions will soon combine with electrons to form neutral atoms. Heavier elements are produced later as a result of astrophysical phenomena. Other than ^7Li, the observed abundances of these elements agree quite well with theoretical computations. As we shall later review, this information is sufficiently accurate to provide a strong bound on the number of additional relativistic species in thermal equilibrium. BBN begins at a temperature $T \sim 1 - 5\,\mathrm{MeV}$. Generating the appropriate light element abundances requires a sufficient number of baryons. Letting n_B and $n_{\overline{B}}$ denote the respective number densities of baryons and anti-baryons in a comoving volume, BBN requires that the ratio between the net baryon number density and photon number density, n_γ is:

$$\eta_B \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma} \sim 6 \times 10^{-10}. \tag{2.22}$$

Era of radiation/coherent oscillation domination: $T \sim 1\,\mathrm{GeV} \to T_B$. Before the start of BBN, the temperature of the universe is sufficiently hot that the particles of the Standard Model exist in a plasma. In standard cosmological scenarios, it is assumed that some initial amount of baryon asymmetry necessary for BBN has been generated at a temperature T_B. At this point, the particles generated during baryogenesis are in thermal equilibrium in a hot plasma. As the Universe cools and expands, a particle species may fall out of thermal equilibrium.

The abundance of these species is particularly important for cosmology. The essential point is that each frozen out species will contribute to the present matter abundance $\Omega_M h^2$. In particular, because the observed dark matter abundance is given by $\Omega_{\mathrm{DM}} h^2 \sim 0.1$, each species must satisfy the bound:

$$\Omega_i h^2 \leq 0.1. \tag{2.23}$$

When this inequality is not satisfied, it is common to say that the species "overcloses" the Universe. As explained in [22], the more precise statement is that when $\Omega_i h^2$ is too large, the predicted value of h would not be in accord with observation.

[2] As reviewed in [22], for example, astrophysical processes also account for a fraction of the ^4He abundance.

In extensions of the standard cosmology, additional dynamics beyond those associated with the freeze out of various species may also play an important role. For example, at temperatures above the start of BBN, scalar fields undergoing coherent oscillations can come to dominate the energy density of the Universe. When such particles decay, they can release significant amounts of entropy into the Universe. This has the effect of diluting the abundance of various frozen out species. In addition, the decay products of such fields can also increase the relic abundance of certain species.

Era of baryon asymmetry creation: $T \sim T_B \to T^0_{\mathrm{RH}}$. At a temperature T_B, the baryon asymmetry requisite for successful BBN is assumed to be created. As found by Sakharov, generating an appropriate baryon asymmetry requires that first, the accidental $\mathrm{U}(1)_B$ baryon symmetry of the Standard Model must be broken, and further, that both C and CP must be violated. In addition, the baryon asymmetry must be generated due to some out of equilibrium process. In order to achieve the required value of η_B, there must be an additional source of CP violation beyond that which is present in the Standard Model. In scenarios where T_B is close to the GUT scale, off diagonal gauge bosons of the GUT group can potentially generate the required values of η_B. Lower temperatures for T_B are also possible depending on the particular mechanism in question. For example, in leptogenesis, an asymmetry in lepton number is converted to an asymmetry in baryon number via sphaleron processes. In these examples, heavy right-handed Majorana neutrinos decay to Standard Model particles, generating the initial lepton number asymmetry so that the associated temperature T_B is roughly given by the Majorana mass of the right-handed neutrinos. An important consequence of this is that the initial reheating temperature T^0_{RH} must satisfy the inequality $T^0_{\mathrm{RH}} \gtrsim \min(M_{\mathrm{maj}})$, where M_{maj} is shorthand for the Majorana masses of the heavy right-handed neutrinos. As we will review later, this can lead to a certain amount of tension in many supersymmetric models which typically aim to lower T^0_{RH} to avoid overproduction of gravitinos. Lower values for T_B are also possible in less standard leptogenesis scenarios, as well as in models where the coherent oscillation of a field generates a large lepton or baryon number. This latter scenario, known as the Affleck-Dine scenario is particularly attractive in models where other cosmological constraints require T_B to be relatively low compared to the value required for other baryon asymmetry scenarios.

Era of speculation: $T \sim T^0_{\mathrm{RH}} \to M_{\mathrm{PL}}$. Above temperatures where the baryon asymmetry is created, there is some initial temperature T^0_{RH} corresponding to the "reheating" (RH) of the Universe. Below this temperature, radiation domination commences. This temperature is typically associated with the end of some era of where some high scale dynamics generates the required density perturbations and flatness of the present Universe. We stress that below the temperature T^0_{RH}, the primary mechanism which sets these initial conditions, which could originate from a mechanism such as inflation or string gas cosmology is inconsequential for the analysis of the paper. In this way, in adhering to the philosophy of [1–5] these issues can be deferred to a later stage of analysis.

2.3 Thermodynamics in the early universe

In the previous subsection we provided a rough sketch for the evolution of the standard cosmology. We now discuss in greater detail the thermodynamics of the Universe, and in particular, review the computation of the relic abundances for hot and cold relics.

2.3.1 Equilibrium thermodynamics

The expanding Universe corresponds to the stage on which the interactions of a given particle physics model will play out. We now summarize some basic features of equilibrium thermodynamics in the early Universe. Much of the following discussion is explained in lucid detail in chapter 3 of [22]. Our aim here is to give a rough intuitive summary of the various formulae which will be important in later discussions.

Assuming that some high scale dynamics sets the initial temperature of the Universe, which we denote by T^0_{RH}, we can follow the subsequent evolution of a cosmological model. At sufficiently high temperatures, various particle species will be in thermal equilibrium. The corresponding interaction rates Γ_{int} within the thermal bath are specified by the collision time for the particle species in question so that:

$$\Gamma_{\mathrm{int}} \sim n_i \langle \sigma_i v_i \rangle \tag{2.24}$$

where $\langle \sigma_i v_i \rangle$ denotes the thermally averaged cross section for the species and n_i denotes its number density. In the limit where the temperature is much greater than the chemical potential, the number density, and energy density of a relativistic species ($m \gg T$) are given by:

$$n_{\mathrm{rel}} \sim T^3 \tag{2.25}$$
$$\rho_{\mathrm{rel}} \sim T^4 \tag{2.26}$$

for a relativistic species. In the other limit where the chemical potential μ dominates, the above formula holds with T replaced by μ. For a non-relativistic species of mass m, the number and energy density are:

$$n_{\text{n-rel}} \sim (mT)^{3/2} \exp\left(-\frac{m-\mu}{T}\right), \tag{2.27}$$
$$\rho_{\text{n-rel}} \sim m n_{\text{n-rel}}. \tag{2.28}$$

Finally, the entropy density s of the thermal bath is primarily determined by the interactions of the relativistic species: and is given by:

$$s \sim g_{*S}(T) T^3, \tag{2.29}$$

where here,

$$g_{*S}(T) \equiv \sum_{\mathrm{bose}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\mathrm{fermi}} g_i \left(\frac{T_i}{T}\right)^3 \tag{2.30}$$

with g_i the number of internal degrees of freedom associated to a given species, so that for example, an electron and positron both have $g_{e^+} = g_{e^-} = 2$. Further, T_i denotes the actual temperature of the given species, which in general may differ from T. Nevertheless, this distinction is largely unimportant when a given species is in equilibrium with the background bath. It is also convenient to introduce a count of the total number of relativistic species defined as:

$$g_*(T) \equiv \sum_{\text{bose}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\text{fermi}} g_i \left(\frac{T_i}{T}\right)^4 . \tag{2.31}$$

We note that at high temperatures, $T_i \sim T$, and $g_*(T)$ and $g_{*S}(T)$ may be used interchangeably. In the high temperature limit where all degrees of freedom are relativistic, the value of g_* in the Standard Model and MSSM are respectively:

$$g_*(SM) = 106.75 \tag{2.32}$$

$$g_*(MSSM) = 228.75. \tag{2.33}$$

In the present era, the net energy and entropy density are given, for example, in appendix A of [22]:

$$\rho_{c,0} \sim (8.1 \times 10^{-47})\, h^2 \text{ GeV}^4 \tag{2.34}$$

$$s_0 \sim 2.3 \times 10^{-38} \text{ GeV}^3 \tag{2.35}$$

where in the above, the subscript 0 reflects the evaluation of this quantity at present times.

The connection between the expansion of the Universe and the temperature of the thermal bath follows from the fact that the total entropy in a co-moving volume:

$$S \propto \left(g_{*S}T^3\right) \cdot a^3 = \text{constant}. \tag{2.36}$$

As a result, the temperature of the universe evolves as:

$$T \propto g_{*S}^{-1/3} a^{-1}. \tag{2.37}$$

Finally, in an era of radiation domination, the fact that the energy density scales as $\rho \propto g_* T^4$, in tandem with the second Friedmann equation (2.6), implies that the Hubble parameter is related to the temperature as:

$$H^2 \sim g_* \frac{T^4}{M_{\text{PL}}^2}. \tag{2.38}$$

This relation will be quite important when we discuss the decoupling of thermal relics during an era of radiation domination.

2.3.2 Relics and decoupling

In the above, we have implicitly assumed that all species in question remain in thermal equilibrium. In this subsection we review what happens when a species decouples from this thermal bath. Much of the material of this subsection is reviewed in greater detail in chapter 5 of [22], and we refer the interested reader there for further discussion.

As the Universe expands, the thermal bath cools, and a given species may decouple. This occurs when the associated comoving volume a^3 becomes too large to allow efficient interactions, so that the $i^{\rm th}$ species "freezes out" at the temperature T_i^f implicitly defined by:

$$H(T_i^f) \sim n_i \langle \sigma_i v_i \rangle . \tag{2.39}$$

More precisely, the evolution of the number density n_i as a function of t is given by the Boltzmann equations in the presence of a dissipation term which accounts for the overall expansion of the Universe:

$$\frac{dn_i}{dt} + 3Hn_i = C_i, \tag{2.40}$$

The left hand side corresponds to the time evolution of the number density, and C_i is determined by the reaction rates of the thermal bath which can generate the $i^{\rm th}$ species. The principle of detailed balance implies that C_i is given by:

$$C_i = \langle \sigma_i v_i \rangle \left(n_{i,\rm EQ}^2 - n_i^2 \right) , \tag{2.41}$$

where $n_{i,\rm EQ}$ denotes the equilibrium number density of the $i^{\rm th}$ species.

After a species is no longer in contact with the thermal bath, its number density redshifts with the expansion of the Universe, scaling as a^{-3}. On the other hand, returning to equation (2.36), it also follows that when the entropy remains constant within a comoving volume, that the entropy density s will also scale as a^{-3}. In determining the relic abundance associated with a frozen out species, it is therefore convenient to introduce the yield:

$$Y_i \equiv \frac{n_i}{s}, \tag{2.42}$$

which modulo subtleties connected to changes in the entropy, remains constant after the $i^{\rm th}$ species has frozen out. During an era of radiation domination $t \propto T^{-2}$, and the Boltzmann equation for the yield attains the form:

$$\frac{dY_i}{dx_i} = \frac{s \langle \sigma_i v_i \rangle}{H(m_i)} \left(Y_{i,\rm EQ}^2 - Y_i^2 \right) , \tag{2.43}$$

where in the above, we have introduced the parameter:

$$x_i = \frac{m_i}{T}. \tag{2.44}$$

After a species has frozen out, the left hand side of equation (2.43) is to leading order, negligible.[3] Hence, the yield at the time of freeze out $Y_{i,\infty}$ is given by $Y_{i,\rm EQ}$ evaluated at the freeze out temperature:

$$Y_{i,\infty} = Y_{i,\rm EQ}(x_i^f). \tag{2.45}$$

Using the yield, we can determine the relic abundance of the $i^{\rm th}$ species. The key point is that because the yield does not change after freeze out, the number density at present times is given by:

$$n_{i,0} = s_0 \cdot Y_{i,\infty}. \tag{2.46}$$

[3]See chapter 5 of [22] for a more precise discussion based on integrating the Boltzmann equations.

As a consequence, the relic abundance is:

$$\Omega_i h^2 = \frac{\rho_{i,0} h^2}{\rho_{c,0}} = \frac{m_i n_{i,0} h^2}{\rho_{c,0}} = \frac{s_0 h^2}{\rho_{c,0}} m_i Y_{i,\infty} \sim 2.8 \times 10^8 \text{ GeV}^{-1} \cdot m_i Y_{i,\infty}, \tag{2.47}$$

where in the final equality, we have used the explicit values of $\rho_{c,0}$ and s_0 given by equations (2.34) and (2.35).

The actual yield of the i^{th} species strongly depends on whether it is relativistic, or only semi-relativistic at the time of freeze out. In the former case, the freeze out temperature is far above the mass of the given particle, so that it is appropriate to use the number density of equation (2.25). In the latter case, the mass may be comparable to, or larger than the freeze out temperature, in which case the number density is given by equation (2.27). Restoring all numerical factors, and using the value of the entropy density in equation (2.29), the yield at the time of freeze out for a relativistic species is:

$$Y^{\text{rel}}_{i,\infty} = \frac{n^{\text{rel}}_{i,\infty}}{s(T^f_i)} \sim \frac{g_{\text{eff}}}{g_{*S}(x^f_i)}, \tag{2.48}$$

where in the above, $g_{\text{eff}} = g$ for bosons and $g_{\text{eff}} = 3g/4$ for fermions, with g the number of degrees of freedom associated with the given species.

The evaluation of the yield in the case of a species which is at most semi-relativistic at freeze out is somewhat more involved. Nevertheless, for our present purposes, the main point is that in a rough approximation, the number density is given by evaluating the equilibrium number density at the temperature of decoupling. Returning to the freeze out condition of equation (2.39), the number density at freeze out is:

$$n^{\text{n-rel}}_{i,\infty} \sim \frac{H(T^f_i)}{\langle \sigma_i v_i \rangle} \sim g_*^{1/2}(T^f_i) \frac{m_i^2}{M_{\text{PL}}} \frac{(x^f_i)^2}{\langle \sigma_i v_i \rangle}. \tag{2.49}$$

Dividing by the entropy density at the time of freeze out, the yield is therefore:

$$Y^{\text{n-rel}}_{i,\infty} = \frac{n^{\text{n-rel}}_{i,\infty}}{s(T^f_i)} \sim \frac{1}{M_{\text{PL}}} \frac{g_*^{1/2}(T^f_i)}{g_{*S}(T^f_i)} \frac{x^f_i}{m_i \langle \sigma_i v_i \rangle}. \tag{2.50}$$

Plugging back into equation (2.47), and restoring all numerical factors, the relic abundance is then given by:

$$\Omega^{\text{rel}}_i h^2 \sim 8 \times 10^{-2} \cdot \frac{g_{\text{eff}}}{g_{*S}(x^f_i)} \left(\frac{m_i}{\text{eV}}\right) \tag{2.51}$$

$$\Omega^{\text{n-rel}}_i h^2 \sim 10^9 \cdot \frac{\text{GeV}^{-1}}{M_{\text{PL}}} \frac{g_*^{1/2}(x^f_i)}{g_{*S}(x^f_i)} \frac{x^f_i}{\langle \sigma_i v_i \rangle}. \tag{2.52}$$

As a point of terminology, relics which decouple when they are relativistic are often called "hot", whereas relics which decouple when they are non-relativistic are called "cold". An important feature of the cold relic density is that it is inversely proportional to the thermally averaged cross section $\langle \sigma_i v_i \rangle$. Assuming that $\langle \sigma_i v_i \rangle \sim \alpha^2/M^2$ for α a fine structure constant on the order of $\sim 1/50$, in a model such as the MSSM or Standard Model where $g_* \sim 100$, a suggestive feature of the above formula is that $\Omega_i^{\text{rel}} h^2 \sim 0.1$ when:

$$M \sim 1 \text{ TeV}. \tag{2.53}$$

An important caveat to the above computations is that it implicitly assumes that the Universe starts at a high enough temperature that the species in question is in thermal equilibrium, and then falls out of equilibrium. Indeed, it is in principle also possible to consider scenarios where the production of a given species is truncated because of T_{RH}^0 being lower than the freeze out temperature of the species. Although somewhat ad hoc, this is one mechanism which has often been invoked to avoid over-production of gravitinos in models where this particle is the LSP.

2.4 The gravitino and its consequences

In the context of F-theory GUTs, the gravitino corresponds to the LSP. Due to the fact that R-parity is typically preserved in such models, the gravitino is stable, and can potentially correspond to a cosmological relic. For example, in the context of high scale gauge mediation scenarios, the gravitino can have a mass as high as 1 GeV, although in the specific context of F-theory GUTs, this value is somewhat lower at $10-100$ MeV. In this subsection we review the fact that in many models, the gravitino relic abundance can overclose the Universe. Indeed, one of the aims of the present paper is to explain how F-theory GUTs naturally solve this "problem".

2.4.1 Freeze out of the gravitino

Because it interacts so weakly with other particles, the freeze out temperature $T_{3/2}^f$ of the gravitino is typically quite high. To estimate the value of $T_{3/2}^f$, we first clarify how the gravitino interacts with the thermal bath of MSSM particles. Following the discussion in for example [24], after supersymmetry is broken, the longitudinal mode of the gravitino $\psi_{3/2}^{\mu}$ eats the spin 1/2 Goldstino mode, ψ associated with supersymmetry breaking so that:

$$\psi_{3/2}^{\mu} \sim \frac{1}{m_{3/2}} \partial^{\mu} \psi. \tag{2.54}$$

Labeling the bosonic component first, given a chiral multiplet (ϕ, χ) or vector multiplet (A_{μ}, λ) the Goldstino mode couples to these fields through the associated supercurrent so that the gravitino Lagrangian density contains the terms [25]:

$$L_{3/2} \supset \frac{i m_{\lambda}}{m_{3/2} \cdot M_{\text{PL}}} [\gamma^{\mu}, \gamma^{\nu}] \overline{\psi} \lambda F_{\mu\nu} + \frac{m_{\chi}^2 - m_{\phi}^2}{m_{3/2} \cdot M_{\text{PL}}} \overline{\psi} \lambda \phi^* + h.c., \tag{2.55}$$

where in the above, the m's denote the masses of various particles, the γ's are the usual Dirac matrices, and we have dropped various constants which are not crucial for the discussion to follow. Letting m denote the characteristic mass scale associated with the mass splitting between members of a given supermultiplet, it follows that the relevant cross section for the gravitino is of the form:

$$\sigma_{3/2} \sim \frac{1}{M_{\rm PL}^2}\left(\frac{m}{m_{3/2}}\right)^2 . \tag{2.56}$$

When in equilibrium, the primary thermal production mechanism for gravitinos is given by the conversion of particles of supersymmetric QCD into gravitinos via processes of the form:

$$AB \to C\psi_{3/2}, \tag{2.57}$$

where here, A, B, C are shorthand for quarks, squarks, gluinos and gluons so that:

$$\sigma_{3/2} \sim \frac{1}{M_{\rm PL}^2}\left(\frac{m_{\widetilde{g}}}{m_{3/2}}\right)^2 , \tag{2.58}$$

where $m_{\widetilde{g}}$ is the mass of the gluino. We refer the interested reader to [24, 26, 27] for a complete list of interactions and the detailed form of the corresponding amplitudes.

Returning to cosmological considerations, the gravitino roughly freezes out at the temperature $T^f_{3/2}$ defined by:

$$H(T^f_{3/2}) \sim n_{3/2} \left\langle \sigma_{3/2} v_{3/2} \right\rangle . \tag{2.59}$$

Precisely because the gravitino only interacts quite weakly with the background thermal bath, it decouples when it is still relativistic. For this reason, it is appropriate to use the relation $n_{3/2} \sim T^3$ for a relativistic species. Furthermore, because the decoupling of the gravitinos happens at high temperatures, this decoupling occurs in an era when radiation dominates the energy density of the Universe. We note in passing that after the gravitino decouples, there could be a transition to a more exotic epoch where matter, or the coherent oscillation of a field dominates the energy density of the Universe. Using the relation between temperature and the Hubble parameter in an era of radiation domination provided by equation (2.38):

$$H^2 \sim g_* \frac{T^4}{M_{\rm PL}^2} \tag{2.60}$$

with g_* the total number of relativistic degrees of freedom, it follows that the freeze out temperature satisfies:

$$T^f_{3/2} \sim g_*^{1/2} M_{\rm PL} \left(\frac{m_{3/2}}{m_{\widetilde{g}}}\right)^2 , \tag{2.61}$$

where in the above, we have set $v_{3/2} \sim 1$, as appropriate for a relativistic species. Including all appropriate numerical factors and performing a more precise estimate based on integrating the Boltzmann equations yields a value $T^f_{3/2} \sim 10^{10} - 10^{11}\,\text{GeV}$ for a gravitino of mass $m_{3/2} \sim 10\,\text{MeV}$ [28, 29]. Indeed, in comparing the overall gravitino relic abundance obtained for $T^0_{\rm RH} < T^f_{3/2}$ (a computation we will shortly review) with the value in the opposite regime where $T^0_{\rm RH} > T^f_{3/2}$, continuity of the gravitino relic abundance across this interpolation yields:

$$T^f_{3/2} \sim 2 \times 10^{10}\ \text{GeV} \cdot \left(\frac{m_{3/2}}{10\ \text{MeV}}\right)^2 \left(\frac{1\ \text{TeV}}{m_{\widetilde{g}}}\right)^2 . \tag{2.62}$$

JHEP04(2010)054

2.4.2 Gravitino relic abundance

Having specified the temperature at which the gravitino freezes out, in this subsection we determine the corresponding relic abundance. As in previous subsections, at this point we will not assume that any late decaying field dilutes the total entropy of the Universe. Assuming that the initial reheating temperature $T^0_{\rm RH}$ is greater than the freeze out temperature, the formulae for the relic abundance of a "hot" species given by equation (2.51) is applicable so that:

$$T^0_{\rm RH} > T^f_{3/2} : \Omega^T_{3/2}h^2 \sim 8\times 10^{-2}\cdot \frac{g_{\rm eff}}{g_{*S}(x^f_{3/2})}\left(\frac{m_{3/2}}{\rm eV}\right), \tag{2.63}$$

where in the above, $\Omega^T_{3/2}$ denotes the fact that these gravitinos are thermally produced. Since the gravitino decouples at such high temperatures, the value of $g_{*S}(x^f_i)$ is given by the total number of degrees of freedom in the MSSM. Including all relevant numerical factors, the resulting relic abundance of gravitinos is [30]:

$$\Omega^T_{3/2}h^2 \sim \frac{m_{3/2}}{2\ \rm keV}. \tag{2.64}$$

A perhaps distressing feature of this formula is that for a gravitino of mass $m_{3/2} \geq 0.2$ keV, the relic abundance would appear to overclose the Universe!

As the above analysis shows, if the MSSM thermal bath starts out at a very high temperature and only cools to the gravitino freeze out temperature at some later stage of expansion, there is a risk that the abundance of gravitinos could overclose the Universe. For this very reason, it is common in the literature to consider scenarios where thermal production of gravitinos has been truncated by lowering the initial reheating temperature $T^0_{\rm RH}$ below $T^f_{3/2}$.

To determine how low $T^0_{\rm RH}$ must be in order to avoid an over-production of gravitinos, we next repeat our analysis of the freeze out temperature of a species detailed in section 2.3.2, but now in the more general case where the start of thermal production commences either above or below the freeze out temperature of the gravitinos. The main change from our previous analysis is that the relic abundance is now determined by the yield at the temperature $T^0_{\rm RH}$, rather than the freeze out temperature. As throughout this review section, our aim here is to give a rough derivation of this formula. More precise derivations may be found for example in [24, 26, 27].

To estimate the abundance of thermally produced gravitinos, we again use the fact that $Y^T_{3/2} = n^T_{3/2}/s$ is roughly constant after the initial production of gravitinos. Here, it is important to note that in principle, the initial reheating temperature $T^0_{\rm RH}$ can either be greater than, or less than the temperature at which gravitinos freeze out. In the latter case, the thermal bath of MSSM particles will begin producing gravitinos up until the temperature at which they freeze out. On the other hand, in the latter scenario, the scattering processes described above will convert MSSM particles into gravitinos at a temperature, $T^0_{\rm RH}$ which immediately freeze out. In this case, the yield of gravitinos is given as:

$$Y^T_{3/2}\left(T^0_{\rm RH}\right) = \frac{n_{3/2}}{s} \sim \frac{C_{3/2}(T^0_{\rm RH})}{H(T^0_{\rm RH})s(T^0_{\rm RH})}, \tag{2.65}$$

where the final estimate follows from the Boltzmann equation (2.40). In this case, the equilibrium number density appearing in $C_{3/2}$ as in equation is given by the number density of the background radiation $n_r(T^0_{\rm RH})$ so that:

$$C_{3/2}(T^0_{\rm RH}) \sim \left\langle \sigma_{3/2} v_{3/2} \right\rangle n_r^2(T^0_{\rm RH}). \tag{2.66}$$

Using the relation $n_r(T^0_{\rm RH}) = s(T^0_{\rm RH})$, the yield is:

$$Y^T_{3/2}\left(T^0_{\rm RH}\right) \sim g_*^{1/2}(T^0_{\rm RH}) \left\langle \sigma_{3/2} v_{3/2} \right\rangle M_{\rm PL} T^0_{\rm RH}. \tag{2.67}$$

Utilizing our expression for the cross section in equation (2.58), we find:

$$Y^T_{3/2}\left(T^0_{\rm RH}\right) \sim g_*^{1/2}(T^0_{\rm RH}) \frac{T^0_{\rm RH}}{M_{\rm PL}} \left(\frac{m_{\tilde g}}{m_{3/2}}\right)^2. \tag{2.68}$$

As a consequence, the relic abundance when $T^0_{\rm RH} < T^f_{3/2}$ is:

$$T^0_{\rm RH} < T^f_{3/2} : \Omega^T_{3/2} h^2 \sim \left(\frac{s_0}{\rho_{c,0}} h^2\right) g_*^{1/2}(T^0_{\rm RH}) \frac{T^0_{\rm RH}}{M_{\rm PL}} \frac{m^2_{\tilde g}}{m_{3/2}}. \tag{2.69}$$

Restoring all numerical factors as in [27]:

$$T^0_{\rm RH} < T^f_{3/2} : \Omega^T_{3/2} h^2 \sim 2.7 \times 10^3 \cdot \left(\frac{T^0_{\rm RH}}{10^{10}\ \rm GeV}\right) \left(\frac{10\ \rm MeV}{m_{3/2}}\right) \left(\frac{m_{\tilde g}}{1\ \rm TeV}\right)^2. \tag{2.70}$$

This is to be contrasted with the relic abundance of gravitinos obtained in the case where $T^0_{\rm RH} > T^f_{3/2}$, where essentially the same formula applies with $T^0_{\rm RH}$ replaced by the freeze out temperature $T^f_{3/2}$. Defining:

$$T^{\rm min}_{3/2} \equiv \min(T^0_{\rm RH}, T^f_{3/2}), \tag{2.71}$$

the gravitino relic abundance in either case is given by:

$$\Omega^T_{3/2} h^2 \sim 2.7 \times 10^3 \cdot \left(\frac{T^{\rm min}_{3/2}}{10^{10}\ \rm GeV}\right) \left(\frac{10\ \rm MeV}{m_{3/2}}\right) \left(\frac{m_{\tilde g}}{1\ \rm TeV}\right)^2. \tag{2.72}$$

Note that when $T^0_{\rm RH} > T^f_{3/2}$, substituting $T^f_{3/2}$ of equation (2.61) into the above relation reproduces the earlier form of the gravitino relic abundance given by equation (2.63):

$$T^0_{\rm RH} > T^f_{3/2} : \Omega^T_{3/2} h^2 \sim \frac{m_{3/2}}{2\ \rm keV}. \tag{2.73}$$

Equation (2.72) illustrates the general puzzling feature of models with a light, stable gravitino. On the one hand, particle physics considerations have a priori nothing at all to do with the value of $T^0_{\rm RH}$. On the other, the overclosure constraint:

$$\Omega_{3/2} h^2 \leq 0.1 \tag{2.74}$$

is badly violated for $m_{3/2} \geq 2$ keV when $T^0_{\rm RH} > T^f_{3/2}$, and when $T^0_{\rm RH} < T^f_{3/2}$ imposes the stringent condition:

$$T^0_{\rm RH} \leq 10^6 \text{ GeV} \cdot \left(\frac{m_{3/2}}{10 \text{ MeV}}\right), \tag{2.75}$$

when the gluino has mass $m_{\widetilde{g}} \sim 1\,\text{TeV}$.

One of the remarkable features which we shall find in section 4 is that these sharp bounds are significantly weaker in F-theory GUT models. In fact, taking the most natural range of parameters dictated from purely particle physics considerations, *we find that this constraint is completely absent in F-theory GUTs*! This is due to the fact that the decay of the saxion can release a significant amount of entropy, diluting the relic abundance of the gravitino.

2.5 Cosmological moduli and their consequences

In the previous subsection we alluded to the crucial role which the decay of the saxion can play in F-theory GUT scenarios. Here, we review the main effects of late-decaying moduli for cosmology. To begin, we clarify our nomenclature for "moduli". In the cosmology literature, it is common to refer to any scalar field which has a nearly flat effective potential as a "modulus". Prominent examples in the context of supersymmetric models are field directions which are massless in the limit where supersymmetry is restored. In the context of string constructions, however, moduli fields typically refer to deformations of the metric or a given vector bundle which can in principle be stabilized by high scale supersymmetric dynamics. To distinguish these two notions of "moduli" we shall always refer to fields which develop a mass due to supersymmetry breaking effects as "cosmological moduli". In this section we show that the coherent oscillation and subsequent decay of cosmological moduli can have important consequences for cosmology.

Cosmological moduli are commonly thought to pose significant problems for cosmology. Indeed, as we now review, such late decaying fields if present can come to dominate the energy density of the Universe. If such fields decay too late, they will disrupt BBN. On the other hand, there is also the potential for such moduli to resolve various problems via their decays. This fact in particular will prove important when we turn to the cosmology of F-theory GUTs.

By definition, the effective potential for cosmological moduli are quite flat. After the initial reheating of the Universe ends at a temperature $T^0_{\rm RH}$, the corresponding scalar modes will have a non-zero amplitude, which we denote by ϕ_0. As the Universe cools, these cosmological moduli develop an effective potential, and begin to oscillate about their minima. In particular, whereas in the standard cosmology, radiation dominates in the range of temperatures between $T^0_{\rm RH}$ and the start of BBN $T_{\rm BBN} \sim 10$ MeV, when the initial amplitude of a modulus field is large enough, this contribution can come to dominate the energy density of the Universe. We now review the estimate for determining when this can occur, and also elaborate on some consequences associated with the subsequent decay of such a modulus field.

Letting $V(\phi)$ denote the effective potential for the modulus, the equation of motion for the ϕ field is given by:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{2.76}$$

where the dots above ϕ denote derivatives with respect to time and the prime denotes the derivative with respect to ϕ. For simplicity, we shall consider the special case where $V(\phi) = m_\phi \phi^2/2$. In this case, the energy density stored in the field is given by:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2, \tag{2.77}$$

where in the above m_ϕ denotes the mass of the modulus in question. In the following analysis we shall always assume that the mass m_ϕ is roughly constant as a function of temperature. When we discuss the cosmology of oscillating axions, we will see that temperature effects play a more significant role.

There are in principle two possibilities for when a modulus will begin to oscillate. The first possibility is that below the initial reheating temperature, the effective potential may develop below the initial temperature of reheating, in which case the modulus begins to oscillate during an era of radiation domination. On the other hand, it is also possible that the modulus could begin oscillating during an era above that set by the initial reheating temperature.

Assuming that the modulus begins to oscillate during an era of radiation domination, the temperature at which the Hubble parameter becomes comparable to the mass scale determines the oscillation temperature $T_{\rm osc}^\phi$ so that:

$$H(T_{\rm osc}^\phi) \sim m_\phi. \tag{2.78}$$

Using the radiation domination relation $H \sim g_*^{1/2} T^2/M_{\rm PL}$, the resulting oscillation temperature is:

$$T_{\rm osc}^\phi \sim g_*^{-1/4}\sqrt{m_\phi M_{\rm PL}}. \tag{2.79}$$

Restoring all numerical factors, a more exact analysis yields:

$$T_{\rm osc}^\phi \sim 0.3\sqrt{m_\phi M_{\rm PL}}. \tag{2.80}$$

We shall frequently refer to this temperature as the "oscillation temperature" of the modulus field.

At temperatures where the modulus field begins to oscillate, the energy density ρ_ϕ stored in the modulus field is governed by the initial amplitude of the modulus and the mass of the modulus field so that:

$$\rho_\phi \sim \frac{1}{2}m_\phi^2\phi_0^2. \tag{2.81}$$

On the other hand, the energy density stored in the background radiation $\rho_r \sim T^4$, so that at the era when the modulus field begins to oscillate, we have:

$$(\rho_r)_{\rm osc} \sim (T_{\rm osc}^\phi)^4. \tag{2.82}$$

JHEP04(2010)054

Combining this relation with equation (2.79), it follows that the ratio of ρ_ϕ and ρ_r at the temperature $T^\phi_{\rm osc}$ is:

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\phi,osc} \sim \frac{\phi_0^2}{M_{\rm PL}^2}. \tag{2.83}$$

An interesting feature of this formula is that ϕ_0, rather than m_ϕ appears.

Following for example [31], we now determine the conditions required for a modulus field to dominate the energy density. For simplicity, we consider a scenario where a single modulus field undergoes coherent oscillation. This scenario can be generalized to situations where multiple fields oscillate and can all contribute a substantial portion of the overall energy density of the Universe.

Assuming that the Universe is in an era of radiation domination, the transition to a modulus dominated energy density can occur provided the energy density stored in the modulus, ρ_ϕ becomes comparable to the background radiation ρ_r. In other words, an era of modulus domination commences at a temperature $T^\phi_{\rm dom}$ where:

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\phi,dom} \sim 1. \tag{2.84}$$

Assuming that the oscillation of the modulus eventually dominates the energy density at a temperature $T^\phi_{\rm dom}$, the scaling behavior $\rho_\phi \propto T^3$ and $\rho_r \propto T^4$ translates into the condition:

$$T^\phi_{\rm dom}\left(\frac{\rho_\phi}{\rho_r}\right)_{\phi,dom} = T^\phi_{\rm osc}\left(\frac{\rho_\phi}{\rho_r}\right)_{\phi,osc}. \tag{2.85}$$

Combined with equation (2.83), the temperature $T^\phi_{\rm dom}$ is given by:

$$T^\phi_{\rm dom} \sim \frac{\phi_0^2}{M_{\rm PL}^2} T^\phi_{\rm osc} \sim \frac{\phi_0^2}{M_{\rm PL}^2}\sqrt{m_\phi M_{\rm PL}}. \tag{2.86}$$

More generally, in scenarios where $T^\phi_{\rm osc}$ is either greater than or less than $T^0_{\rm RH}$, $T^\phi_{\rm dom}$ is given by:

$$T^\phi_{\rm dom} \sim \frac{\phi_0^2}{M_{\rm PL}^2}\min(T^\phi_{\rm osc}, T^0_{\rm RH}). \tag{2.87}$$

In the above estimate, we have implicitly assumed that the modulus field is sufficiently long lived that it can dominate the energy density. It is also possible, however, that the modulus field may decay before, or after an era of modulus domination is able to commence. For example, when $\phi_0/M_{\rm PL}$ is sufficiently low, the resulting value of $T^\phi_{\rm dom}$ may correspond to a timescale which is longer than the lifetime of ϕ. In such a scenario, the decay of ϕ will occur in an era of radiation domination.

The timescale for the decay of the ϕ particle is given by the inverse of Γ_ϕ, the decay rate for ϕ. The modulus ϕ will either decay during an era of radiation domination, or at the end of ϕ domination. In the latter case, we shall assume that the Universe then transitions back to an era of radiation domination. In both cases, the corresponding temperature $T^\phi_{\rm decay}$ at the time of decay therefore scales with $t^\phi_{\rm decay} = \Gamma_\phi^{-1}$ as:

$$\sqrt{t^\phi_{\rm decay}} \propto (T^\phi_{\rm decay})^{-1}, \tag{2.88}$$

which holds when radiation dominates the energy density of the Universe. The relation:

$$H(T_{\text{decay}}^{\phi}) \sim \Gamma_{\phi} \tag{2.89}$$

therefore implies:

$$T_{\text{decay}}^{\phi} \sim g_*^{-1/4} \sqrt{\Gamma_{\phi} M_{\text{PL}}}. \tag{2.90}$$

Restoring all numerical factors, a more exact analysis yields:

$$T_{\text{decay}}^{\phi} \sim 0.5 \sqrt{\Gamma_{\phi} M_{\text{PL}}}. \tag{2.91}$$

We note that if Γ_{ϕ} is sufficiently small, this can disrupt the standard predictions of BBN, significantly altering standard cosmology. As we now explain, in situations where ϕ dominates the energy density of the Universe, it is also common to refer to this temperature as the "reheating temperature" of ϕ, and we shall therefore also use the notation:

$$T_{\text{RH}}^{\phi} \equiv T_{\text{decay}}^{\phi} \sim 0.5 \sqrt{\Gamma_{\phi} M_{\text{PL}}}. \tag{2.92}$$

In order for the Universe to enter an era of modulus domination, the three temperatures T_{decay}^{ϕ}, T_{dom}^{ϕ} and T_{RH}^{0} must satisfy the system of inequalities:

$$T_{\text{decay}}^{\phi} < T_{\text{dom}}^{\phi} < \min(T_{\text{osc}}^{\phi}, T_{\text{RH}}^{0}). \tag{2.93}$$

Returning to equations (2.86) and (2.91), this amounts to a condition on the properties of the modulus:

$$\sqrt{\Gamma_{\phi} M_{\text{PL}}} \lesssim \frac{\phi_0^2}{M_{\text{PL}}^2} \sqrt{m_{\phi} M_{\text{PL}}}, \tag{2.94}$$

or:

$$\sqrt{\frac{\Gamma_{\phi}}{m_{\phi}}} \lesssim \frac{\phi_0^2}{M_{\text{PL}}^2}. \tag{2.95}$$

The decay products of the modulus field can also have significant consequences for the cosmology of the Universe. Again, the immediate consequence naturally separates into scenarios where the modulus decays in an era of radiation domination or modulus domination. In the former case, the primary constraint is that the resulting decay products must satisfy the bound (which we shall review later) on the total number of relativistic species, in order to remain in accord with BBN. While a similar constraint also holds in the case where the modulus comes to dominate the energy density of the Universe, another significant effect is the release of entropy into the Universe as a result of the decay of the modulus. This increase in entropy effectively dilutes the total relic abundance of all species produced prior to this decay.

Letting $\Omega_z^{(0)} h^2$ denote the relic abundance of a species z prior to the decay of ϕ, the relic abundance after dilution is then given by:

$$\Omega_z h^2 = D_{\phi} \Omega_z^{(0)} h^2 \equiv \frac{s_{\text{before}}}{s_{\text{after}}} \Omega_z^{(0)} h^2, \tag{2.96}$$

where in the above, s denotes the entropy density, and we have introduced the "dilution factor":

$$D_\phi \equiv \frac{s_{\text{before}}}{s_{\text{after}}}. \tag{2.97}$$

A similar analysis to that utilized in estimating the ratio of energy densities in equation (2.83) yields the following estimate for the dilution factor, which can be found, for example, in [18, 22, 32, 33]:

$$D_\phi \sim \frac{T^\phi_{\text{RH}}}{T^\phi_{\text{dom}}} \sim \frac{M^2_{\text{PL}}}{\phi_0^2} \frac{T^\phi_{\text{RH}}}{\min\left(T^0_{\text{RH}}, T^\phi_{\text{osc}}\right)}. \tag{2.98}$$

In the above, the minimum of T^0_{RH} and T^ϕ_{osc} enters because there are in principal two possible scenarios, where either the temperature of oscillation is larger, or smaller than T^0_{RH}. The essential point is that the lower temperature more strongly determines the resulting entropy density of the Universe denoted by s_{before}, so that it is always appropriate to take the minimum of these two parameters. The release of this large entropy into the Universe effectively "reheats" the Universe.

An implicit assumption of equation (2.98) is that the dilution factor D_ϕ is less than one. Indeed, in scenarios where ϕ decays during an era of radiation domination, the only effect on the history of the Universe may be an increase in the relic abundance of a given species. When D_ϕ is formally greater than one, it follows from equation (2.98) that the temperature T^ϕ_{dom} is less than T^ϕ_{RH}, violating one of the implicit assumptions used to derive our expression for the dilution factor.

2.5.1 The saxion as a cosmological modulus

In supersymmetric models which also solve the strong CP problem via an axion, the other real bosonic degree of freedom of the corresponding chiral supermultiplet, which we shall refer to as the saxion provides an important example of a cosmological modulus. The essential point is that in the limit where supersymmetry is restored, the axion and saxion have the same mass. Thus, just as for any other cosmological modulus, supersymmetry breaking effects provide the dominant contribution to the effective potential of the saxion.

The presence of the saxion is especially significant in models where the scale of supersymmetry breaking is lower than in gravity mediation models. On general grounds, many of the moduli of the string compactification in such cases will have large masses determined by high scale supersymmetric dynamics. Indeed, in such a scenario essentially the only remaining cosmological moduli are those required from particle physics considerations, the saxion being the primary example of such a cosmological modulus. The dynamics of this field in particular will play an important role in the cosmology of F-theory GUT models.

2.6 Oscillations of the axion

In the previous subsection we reviewed the fact that the oscillation of a modulus can alter the evolution of the Universe, leading to an era of modulus domination, as well as an overall dilution of all relic abundances due to the release of entropy into the Universe. In particular, we also reviewed the fact that the saxion, as a component of the axion supermultiplet can play the role of such a cosmological modulus. In this subsection we review the cosmology associated with the axion, focussing in particular on the effects of its oscillation.

We first begin by reviewing some details of the axion. By definition, the QCD axion a couples to the QCD instanton density so that the Lagrangian density for a contains the terms:

$$L_{\text{axion}} = \frac{f_a^2}{2}\left(\partial_\mu a\right)^2 + \frac{a}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma} Tr_{SU(3)_C} F_{\mu\nu} F_{\rho\sigma} \tag{2.99}$$

where here, $F_{\mu\nu}$ denotes the field strength of $SU(3)_C$, and f_a denotes the axion decay constant. The field a corresponds to the Goldstone mode associated with spontaneous breaking of an anomalous global U(1) symmetry and takes values in the interval:

$$-\pi < a < \pi. \tag{2.100}$$

Constraints from supernova cooling impose a lower bound on f_a so that:

$$f_a > 10^9 \text{ GeV}. \tag{2.101}$$

The upper bound on f_a is based on cosmological considerations, and is in general more flexible. One of the purposes of this subsection is to review the derivation of this upper bound.

The effective potential for the axion is generated by QCD instanton effects and can be approximated using the pion Lagrangian, as for example in section 23.6 of [34]:

$$V_{\text{axion}}(a) = m_\pi^2 f_\pi^2 \left(1 - \cos a\right), \tag{2.102}$$

where $m_\pi \sim 130\,\text{MeV}$ is the mass of the pion, and $f_\pi \sim 90\,\text{MeV}$ is the pion decay constant. The mass of the canonically normalized axion is therefore:

$$m_a \sim \frac{m_\pi f_\pi}{f_a} \sim 6 \times 10^{-5} \text{ eV} \cdot \left(\frac{10^{12} \text{ GeV}}{f_a}\right). \tag{2.103}$$

At temperatures $T \gg \Lambda_{\text{QCD}} \sim 0.2\,\text{GeV}$, the mass of the axion depends non-trivially on T, and is given by:

$$m_a(T) = m_a \cdot \left(\frac{\Lambda_{\text{QCD}}}{T}\right)^4. \tag{2.104}$$

The axion is a very long lived particle, and can therefore have consequences for cosmology. Returning to the pion Lagrangian, it can be shown that the primary decay channel of the axion is into two photons. Following section 23.6 of [34], the relative decay rates between $a \to \gamma\gamma$ and $\pi^0 \to \gamma\gamma$ is given by the square of the ratios of the two decay constants, f_a and f_π multiplied by an overall phase space factor proportional to m_a^3/m_π^3. The relative decay rates are therefore:

$$\frac{\Gamma_{a\to\gamma\gamma}}{\Gamma_{\pi^0\to\gamma\gamma}} \sim \left(\frac{f_\pi}{f_a}\right)^2 \left(\frac{m_a}{m_\pi}\right)^3 \sim \left(\frac{f_\pi}{f_a}\right)^5. \tag{2.105}$$

Using the lifetime of π^0 given by $\tau_{\pi^0} \sim 8.4 \times 10^{-17}$ sec, it follows that the lifetime of the axion is:

$$\tau_a \sim 10^{-16}\,\text{sec} \cdot \left(\frac{f_\pi}{f_a}\right)^{-5} \sim 10^{49}\,\text{sec} \cdot \left(\frac{10^{12}\ \text{GeV}}{f_a}\right)^{-5}, \tag{2.106}$$

which is far greater than the current lifetime of the Universe $\sim 10^{18}$ sec.

Because the axion is quite long lived in comparison to cosmological timescales, it can in principle play an important role in cosmology. Even so, due to its small mass and the fact that all couplings of the axion to the Standard Model and MSSM degrees of freedom are suppressed by powers of $1/f_a$, the total relic abundance of axions produced from thermal processes is typically quite small. We refer the interested reader to [22], for example, for further details on such estimates.

The primary cosmological issue connected to the axion is the fact that at high temperatures, the potential for the axion is nearly flat, and the field can easily be displaced from its minimum. Much as for cosmological moduli, the oscillation of the axion can then have consequences for cosmology, appearing as a zero momentum condensate of non-relativistic particles. The corresponding contribution to the overall energy density of the Universe can in principle overclose the Universe, or provide a significant component of the overall dark matter. To this end, we now review the associated relic abundance from coherent oscillation of the axion.[4]

2.6.1 Axionic dark matter

As indicated previously, the axion is nearly massless at high temperatures. By inspection of equation (2.51), the thermal production of axionic relics is very small owing to the small mass of the axion $m_a \sim 10^{-5}\,$eV. For this reason, axionic dark matter is only a viable candidate when produced through some non-thermal mechanism. Precisely because of its small mass, the axion can be displaced from its minimum to a value a_0 such that $-\pi < a_0 < \pi$. Throughout our discussion, we shall assume that this initial displacement is given by roughly the same value in causally disconnected patches of the Universe. We note that in scenarios where $T^0_{\text{RH}} < f_a$, whatever mechanism solves the homogeneity problem will also translate into a uniform value for the initial amplitude in the entire causal patch of the Universe. At higher temperatures where $T^0_{\text{RH}} > f_a$, the axion is still not well-defined, so that once the Universe cools to a temperature below f_a, the initial amplitude of the axion may be different in distinct patches of the Universe. Nevertheless, this simply amounts to replacing the uniform value of the initial amplitude by a rough average over various causal patches.

[4] Although it is beyond the scope of the present paper to present speculations on the evolution of the Universe at temperatures greater than T^0_{RH}, in the context of inflationary models where $T^0_{\text{RH}} < f_a$, quantum fluctuations in the oscillation of the axion of the form $a = \langle a \rangle + \delta a$ associated with oscillation of the axion can in principle induce density perturbations, leading to small variations in the CMBR. These "isocurvature perturbations" occur in models where the fluctuation mode exits the horizon during the expected de Sitter phase and remains frozen in until some time after inflation ends, at which point the mode re-enters the horizon. In the context of models in which inflation occurs at a temperature $T^0_{\text{RH}} < f_a$, this leads to a bound on the reheating temperature of the form: $T^0_{\text{RH}} \lesssim 10^{13}\ \text{GeV} \cdot \left(\Omega_{\text{ax}} h^2\right)^{-1/4} \cdot (f_a/10^{12}\ \text{GeV})^{5/24}$. See for example [22] for further details of isocurvature perturbations.

Once the Universe cools sufficiently, the axion will begin to oscillate, creating a condensate of zero momentum particles. We now proceed to estimate the effective number density of this condensate, and compute the associated relic abundance of axions. Just as in our review of general cosmological moduli, under the assumption that the mass term dominates the effective potential, the equation of motion for the axion is given by:

$$\ddot{a} + 3Ha + m_a^2(T)a = 0 \tag{2.107}$$

where here, we have included the explicit T dependence of m_a given by equation (2.104).

A priori, the axion may begin to oscillate during an era of either radiation or modulus domination. In the latter case, the decay of the modulus can dilute the relic abundance of the axion, so that in principle, the value of the decay constant can be increased [18]. However, when this is not done, a comparison of the relic abundances obtained from oscillation in an era of radiation domination and the "undiluted" relic abundance of the axion obtained from an era of modulus domination are numerically quite similar. For this reason, the diluted relic abundance is typically negligible when the axion starts oscillating before the modulus decays. Since we are interested in the case where the undiluted relic abundances are numerically quite similar anyway, for our present purposes it is sufficient to review the relic abundance computation in the case where the axion begins oscillating during an era of radiation domination.

The axion begins to oscillate at a temperature $T^a_{\rm osc}$ where the mass $m_a(T)$ is comparable to the overall Hubble parameter:

$$H \sim m_a\left(T^a_{\rm osc}\right). \tag{2.108}$$

Assuming that it begins oscillating during an era of radiation domination so that $H \sim g_*^{1/2}T^2/M_{\rm PL}$ it follows that $T^a_{\rm osc}$ is given by:

$$\frac{T^a_{\rm osc}}{\Lambda_{\rm QCD}} \sim \left(\frac{M_{\rm PL}m_a}{g_*^{1/2}\Lambda^2_{\rm QCD}}\right)^{1/6} \sim 10 \cdot \left(\frac{10^{12}\ {\rm GeV}}{f_a}\right)^{1/6}. \tag{2.109}$$

Dropping the weak dependence on f_a, the temperature at which the axion begins to oscillate is therefore given by:

$$T^a_{\rm osc} \sim 10 \cdot \Lambda_{\rm QCD} \sim 1\ {\rm GeV}, \tag{2.110}$$

which is only a few orders of magnitude away from the start of BBN, with $T_{\rm BBN} \sim 2\,{\rm MeV} \sim 10^{-2}\cdot\Lambda_{\rm QCD}$.

We now proceed to determine the relic abundance of the axion. As in our analysis of the cosmological modulus, the energy density stored in the axion when it commences oscillation is given by:

$$\rho_a(T^a_{\rm osc}) \sim \frac{1}{2}m_a^2(T^a_{\rm osc})(f_a a_0)^2. \tag{2.111}$$

Treating the field condensate as a collection of non-relativistic particles at zero momentum, the initial number density is:

$$n_a(T^a_{\rm osc}) \sim \frac{\rho_a(T^a_{\rm osc})}{m_a(T^a_{\rm osc})} \sim \frac{1}{2}m_a(T^a_{\rm osc})(f_a a_0)^2 \sim \frac{1}{2}m_a(f_a a_0)^2\cdot\left(\frac{\Lambda_{\rm QCD}}{T^a_{\rm osc}}\right)^4. \tag{2.112}$$

The yield of axions is therefore:

$$Y_a = \frac{n_a(T^a_{\rm osc})}{s(T^a_{\rm osc})} \sim m_a (f_a a_0)^2 \cdot (T^a_{\rm osc})^{-3} \left(\frac{\Lambda_{\rm QCD}}{T^a_{\rm osc}}\right)^4 . \tag{2.113}$$

Including all relevant numerical factors, a similar analysis to the one already presented yields the final estimate for the axion relic abundance [23]:

$$\Omega_{\rm ax} h^2 \sim a_0^2 \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{7/6} . \tag{2.114}$$

Thus, under circumstances where a_0 is roughly an order one number, it follows that overclosure constraints impose the condition $f_a \lesssim 10^{12}$ GeV, so that axions can in principle comprise a component of dark matter. Nevertheless, this is quite sensitive to the actual value of a_0 so that even when $a_0 \sim 10^{-1}$, the corresponding relic abundance will be negligible. Further, in the event that the axion begins oscillating during an era of modulus domination, the numerical similarity of the two relic abundances for $f_a \sim 10^{12}$ GeV implies that once the effects of dilution are taken into account, in the latter case the relic abundance is always negligible.

2.7 Constraints from BBN

While many extensions of the Standard Model come equipped with potential dark matter candidates, it is also quite important to check that any such extension does not introduce additional elements which conflict with well-established features of the standard cosmology. In this regard, the standard cosmology prediction for the abundances of the light nuclei H^+, D^+ are in excellent agreement with observation, and are in reasonable accord with $^3He^{++}$, $^4He^{++}$. The predicted abundance of ^7Li derived from the standard cosmology appears to reveal a discrepancy between theory and observation. Optimistically speaking, this can be viewed as a potential window into the physics beyond the Standard Model which could potentially alter some of the reaction rates present in standard nucleosynthesis.

One of the most remarkable features of BBN is that the resulting abundances of light elements essentially depend on only the expansion rate of the Universe, and the overall baryon asymmetry:

$$\eta_B \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma}, \tag{2.115}$$

where in the above, n_B, $n_{\overline{B}}$ and n_γ respectively denote the number density of baryons, anti-baryons and photons. Quite remarkably, although the resulting abundances span approximately *nine* orders of magnitude, they are all correctly accounted for when the baryon asymmetry falls within the narrow window:

$$4.7 \times 10^{-10} \lesssim \eta_B \lesssim 6.5 \times 10^{-10}. \tag{2.116}$$

Extensions of the standard cosmology can potentially threaten this result in one of two ways. As we will shortly review, BBN imposes significant limits on the overall expansion rate, and as such, effectively constrains the total number of relativistic species present at the start of BBN. The presence of late-decaying particles can also alter the results of BBN by either destroying, or producing too much of a given light element. For example, in the context of the MSSM, in scenarios with a bino-like NLSP, the decay of this particle into a photon and gravitino can potentially disrupt the production of certain elements through the presence of additional background photons. In certain cases, however, such decays can in fact *improve* the agreement between theory and observation. An important example of this type is the overall abundance of ^7Li, which in the standard cosmology turns out to be a factor of $2-5$ larger than is observed. In the remainder of this subsection, we provide additional details on these two central constraints.

2.7.1 BBN bounds on relativistic species

As reviewed for example in [22], increasing the expansion rate of the Universe leads to an increase in the total amount of ^4He produced by BBN. Due to the connection between the Hubble parameter and the number of relativistic degrees of freedom coupled to the thermal bath:

$$H = \sqrt{\frac{g_*(T)\pi^2}{90M_{\rm PL}^2}}T^2, \tag{2.117}$$

a constraint on the expansion rate translates into a direct bound on the total number of relativistic species. Although it is beyond the scope of the present paper to review the derivation of how production of ^4He translates into a bound on the expansion rate of the Universe, the end result of this calculation is that at the start of BBN, the total number of relativistic species must satisfy:

$$g_*(T \sim \mathrm{MeV}) \leq 12.5. \tag{2.118}$$

where $g_*(T)$ as a function of T is given by equation (2.31). The relativistic degrees of freedom of the Standard Model already nearly saturate this upper bound, and are given by the photon ($g_\gamma = +2$), three species of neutrinos ($g_\nu = 6$) and the electrons and positrons ($g_{e^-} = g_{e^+} = 2$). Because these particles are all in thermal equilibrium, equation (2.31) simplifies to the special case where $T_i = T$ for all species so that:

$$g_*^{\rm SM}(T \sim 1\ \mathrm{MeV}) = 2 + \frac{7}{8}(6+2+2) = 10.75. \tag{2.119}$$

Combined with the strong upper bound provided by (2.118), the general temperature dependence of g_* provided by equation (2.31) leads to the inequality:

$$\sum_{\text{new bose}} g_i \left(\frac{T_i}{1\ \mathrm{MeV}}\right)^4 + \frac{7}{8}\sum_{\text{new fermi}} g_i \left(\frac{T_i}{1\ \mathrm{MeV}}\right)^4 < 1.75. \tag{2.120}$$

Note in particular that even one fermionic species in thermal equilibrium already completely saturates this upper bound. For example, this bound implies that only one additional species of interacting relativistic neutrinos can be included in an extension of the Standard Model, and that in this case, absolutely no additional degrees of freedom can be added without disrupting BBN! This same condition can also be stated as a bound on the total energy density contributed by an additional relativistic species:

$$\left(\frac{\rho_{\text{extra}}}{\rho_r}\right)_{\text{BBN}} \leq \frac{7}{43}, \tag{2.121}$$

where ρ_{extra} denotes the energy density stored in the extra relativistic species.

It is important to qualify that the bound on the number of relativistic species is most stringent in the case of additional degrees of freedom which directly couple to the background thermal bath. For example, this might appear to contradict the possibility of Dirac-like neutrinos, because *if* additional light states happened to be in thermal equilibrium at the start of BBN, inequality (2.120) would be violated.

Additional relativistic species could be present if they decouple at a sufficiently high temperature, which we denote by T_D. Comparing the scale factor dependence in equations (2.16) and (2.17) with the temperature dependence in equations (2.26) and (2.28), it follows that the temperature of a decoupled species i obeys the relation:

$$\text{Decoupled and Relativistic: } a \propto \frac{1}{T_i}. \tag{2.122}$$

On the other hand, species which remain coupled to the thermal bath are more directly sensitive to changes in the number of relativistic species. Indeed, equation (2.37) implies that the overall scaling of the thermal bath evolves as:

$$aT \propto g_{*S}^{-1/3}(T). \tag{2.123}$$

Comparing equations (2.122) and (2.123), the resulting temperature ratio entering inequality (2.120) is:

$$\frac{T_i}{T} \sim \left(\frac{10.75}{g_*(T_D)}\right)^{1/3}. \tag{2.124}$$

In other words, if a species decouples at a sufficiently high temperature, the actual contribution to $g_*(T \sim 1\,\text{MeV})$ will be significantly suppressed.

Returning to the case of right-handed neutrinos mentioned previously, we note that such particles decouple at a temperature T_D such that:

$$g_*(T_D) \geq 106.75, \tag{2.125}$$

where this lower bound corresponds to the number of degrees of freedom of the Standard Model. As a consequence, we obtain the relation:

$$6 \cdot \frac{7}{8}\left(\frac{T_{\nu_R}}{T}\right)^4 \lesssim 0.2, \tag{2.126}$$

so that inequality (2.120) remains intact.

JHEP04(2010)054

2.7.2 BBN and late decaying particles

Given the fact that even crude bounds from BBN tied to the expansion rate of the Universe translate into detailed constraints on the number of relativistic species, it is perhaps not surprising that the reaction rates necessary for generating the correct abundance of light elements from BBN are also quite sensitive to the presence of late decaying particles. On the one hand, this imposes important constraints on potential extensions of the Standard Model, because the abundances of the light nuclei H^+, D^+, T^+, ${}^3He^{++}$, ${}^4He^{++}$ are all in reasonable accord with observation. On the other hand, this also provides a window into new physics, because the standard cosmology appears to predict an abundance of ${}^7Li$ which is too large by a factor of $2-5$ when compared with observation. We refer the interested reader to [35] and references therein for a very recent account of the current bounds on various abundances.

As briefly mentioned above, late decaying particles are possible in certain supersymmetric extensions of the Standard Model. For example, in the context of the MSSM where the effects of supersymmetry breaking are communicated via gauge mediation, the gravitino is the lightest superpartner of the MSSM, and either the bino or stau corresponds to the next to lightest superpartner (NLSP). This is the case of primary interest for F-theory GUTs, and so in the remainder of this subsection we shall therefore restrict attention to this case.

The decay rate of the NLSP into a gravitino and its Standard Model counterpart is determined by the universal coupling of the gravitino to matter provided by equation (2.55). The calculation of the lifetime of the NLSP is reviewed for example, in [36], and leads to the well known result:

$$\tau_{\text{NLSP}} \sim \frac{6 \times 10^{-2}\ \text{sec}}{\kappa} \cdot \left(\frac{m_{\text{NLSP}}}{100\ \text{GeV}}\right)^{-5} \left(\frac{m_{3/2}}{10\ \text{MeV}}\right)^{2}, \tag{2.127}$$

where m_{NLSP} denotes the mass of the NLSP and κ is a model dependent factor which is unity for the case of the stau NLSP, and measures the photino content of the bino in the case of the bino NLSP. The particular normalization for the two masses has been chosen to conform with natural values in the range expected in the specific context of a high-scale gauge mediation model, as is the case in the context of F-theory GUTs. The lifetime of the NLSP is to be compared with the timescale of BBN, which roughly commences at a temperature of $T_{\text{BBN}} \sim 1\,\text{MeV}$, corresponding to the timescale $t \sim 0.2$ s. By inspection of equation (2.127), it follows that in certain situations, the NLSP could potentially decay just before the start of, or even during BBN!

At the most conservative level, the usual results of the standard BBN cosmology can typically be retained if the NLSP decays prior to the start of BBN. Returning to equation (2.127), decreasing $m_{3/2}$ or increasing m_{NLSP} will both decrease the value of τ_{NLSP}. In particular, the fact that the fifth power of m_{NLSP} appears in τ_{NLSP} implies that even very mild adjustments in this value can significantly decrease the lifetime of the NLSP.

Assuming that the NLSP decays during BBN, its decay products could potentially jeopardize the production of the light element abundances, or could bring the abundance of light elements such as ${}^7Li$ into *better* accord with observation. The precise effect of the

NLSP depends on whether it decays to a photon and gravitino as for a bino-like NLSP; or whether it decays to a tau and gravitino, as in the case of a stau NLSP. The impact from a late decaying NLSP depends on details of a particular model, such as the overall abundance of the NLSP prior to the start of BBN. Nevertheless, under reasonable assumptions such that the relative abundance of the NLSP to the overall baryon density is not distorted by dilution effects, it is possible to estimate the production of the light elements due to BBN.

In the case of a bino-like NLSP, the analysis reflected in figure 7 of [37] indicates that when the gravitino has a mass of $10-100\,\text{MeV}$, the main predictions of BBN remain intact. At larger values of the gravitino mass in the range of $\gtrsim 1\,\text{GeV}$, only somewhat large values of the bino mass remain in accord with BBN. Similarly, in the case of a stau NLSP, figure 12 of [37] illustrates that the same range for the gravitino mass remains in accord with BBN with similar constraints for the mass of the stau in the range of larger values of the gravitino mass.

Interestingly, the very recent analysis of [35] also indicates that a gravitino in this same mass range appears to also decrease the overall abundance of ^7Li, bringing the resulting abundance into better agreement with observation. Although a complete analysis of BBN in the context of F-theory GUTs is beyond the scope of the present paper, quite auspiciously, the natural range of parameters of F-theory GUTs suggests a gravitino mass in the range of $10-100\,\text{MeV}$!

3 F-theory GUTs and the axion supermultiplet

In this section we study the axion supermultiplet in F-theory GUT models. To this end, we first briefly review the main features of F-theory GUTs [1–3]. This is followed by a discussion of the axion supermultiplet, and in particular, the interactions of the saxion with the MSSM.

In F-theory GUT models, the singularity type of an elliptically fibered Calabi-Yau fourfold with section over subspaces of the threefold base of complex codimension one, two and three respectively determine the resulting gauge symmetry, matter content and interactions terms of the low energy effective theory. Singularity type enhancements along complex surfaces are interpreted as seven-branes with ADE gauge group. These seven-branes then intersect over Riemann surfaces or "matter curves" where the singularity type enhances by at least one rank. Finally, this enhancement can increase further at points of the compactification, corresponding to the intersection of at least three seven-branes at a single point.

Although perhaps contrary to previous experience with string compactifications, local F-theory GUTs provide a somewhat rigid framework for string based model building. In these models, the existence of a limit where gravity can in principle decouple requires that the GUT group seven-brane must wrap a del Pezzo surface. Moreover, breaking the GUT group to the Standard Model gauge group is remarkably constrained in such models [2, 9].

Next consider the supersymmetry breaking sector of F-theory GUTs. For the purposes of this paper we shall assume that the primary features of the deformation of a minimal gauge mediation scenario developed in [3] are satisfied. The existence of a gauge mediation scenario assumes that most moduli present are stabilized due to high scale supersymmetric dynamics. Much as in [3], our attitude will be that phenomenological constraints on the particle physics content of this class of models should be viewed as imposing interesting restrictions on possible global completions which satisfy these conditions.

We now review some further details of the supersymmetry breaking sector discussed in [3]. Consistent electroweak symmetry breaking requires that the parameter μ of the MSSM superpotential:

$$L_{\text{MSSM}} \supset \int d^2\theta \mu \cdot H_u H_d \tag{3.1}$$

must not be significantly different from the weak scale. If supersymmetry breaking indeed stabilizes the hierarchy between the weak scale and the GUT scale, this naturally suggests that the value of μ should be correlated with the scale of supersymmetry breaking. The effects of supersymmetry breaking can be parameterized in terms of the vev of a GUT group singlet chiral superfield X such that:

$$\langle X \rangle = x + \theta^2 F. \tag{3.2}$$

In [3], some explicit solutions to the μ problem were obtained under the assumption that X localizes on a matter curve which intersects the GUT seven-brane at a point. Integrating out the Kaluza-Klein modes on the X field curve generates a higher dimension operator in the effective theory of the form:

$$L_{\text{MSSM}} \supset \gamma \cdot \int d^4\theta \frac{X^\dagger H_u H_d}{M_X} \tag{3.3}$$

where, as estimated in [2], $M_X \sim 10^{15.5\pm0.5}$ is the Kaluza-Klein scale associated with the curve supporting the X field and $\gamma \sim O(10)$. In order to generate the correct value of the μ term, F must attain the value $\sim 10^{17\pm0.5}\,\text{GeV}^2$. As explained in [3], this value is naturally attained via instanton effects associated with Euclidean three-branes wrapping other complex surfaces of the geometry. As explained in [3], the value of the $B\mu$ term is naturally suppressed relative to μ^2. This is because localization on seven-branes and the presence of a PQ symmetry implies the leading order contribution to $B\mu$ at the messenger scale is from the operator:

$$\int d^4\theta \frac{X^\dagger X X^\dagger H_u H_d}{M_X^3}, \tag{3.4}$$

which induces a $B\mu$ term of order $\mu^2 \cdot (x/M_X)$, which is far smaller than μ^2. $B\mu$ is then generated at lower scales through radiative corrections.

Following the model of [3], the supersymmetry breaking sector is closely tied to the seven-brane associated with the $U(1)_{\text{PQ}}$ gauge symmetry. This gauge theory is anomalous, and as such, instanton contributions can generate contributions to the superpotential of the form:

$$W_{\text{inst}} \supset M_{\text{PQ}}^2 \cdot q \cdot X \tag{3.5}$$

where:

$$q \sim e^{-Vol_{\text{PQ}}}. \tag{3.6}$$

Assuming a fixed value for q, this determines the F-term component of X. As explained in [3], to properly analyze the PQ symmetry breaking sector, it is necessary to treat both q and X as dynamical fields. In that context, it was shown that an appropriate tuning in the Kähler potential for q and the flux-induced FI parameter for the PQ gauge theory is compatible with stabilizing the vev of X at the scale 10^{12} GeV. The Goldstone mode associated with the breaking of the accidental global $U(1)_{PQ}$ symmetry is parameterized by the phase of the gauge invariant operator $q \cdot X$. In particular, the axion is therefore primarily given by the phase of X, with a small contribution from q. The corresponding axion decay constant is then given as:

$$f_a = \sqrt{2}x \sim 10^{12} \text{ GeV}. \tag{3.7}$$

To a large extent, the allowed mediation mechanism which communicates the effects of supersymmetry breaking to the visible sector is dictated by the fact that F/M_{PL} is far below the weak scale so that Planck suppressed operators, and therefore gravity mediated supersymmetry breaking cannot generate viable soft mass terms. By contrast, geometric realizations of minimal gauge mediation scenarios are far more viable in this scenario, and explicit models based on minimal gauge mediation have been discussed in [3] (see also [11]). In minimal gauge mediation (MGMSB), the soft mass terms are completely fixed by the gauge couplings of the MSSM and the ratio $F_X/x \equiv \Lambda$ as:

$$m_{\text{soft}} \sim \frac{\alpha}{4\pi}\frac{F}{x} = \frac{\alpha}{4\pi} \cdot \Lambda \tag{3.8}$$

up to numerical factors associated with the representation content of a given field. Here, α is shorthand for the fine structure constants of the various gauge groups of the Standard Model under which a given superfield may be charged.

As explained in [3], the F-theory GUT actually corresponds to a deformation of minimal gauge mediation. Indeed, precisely because the X field localizes on a matter curve, it will be charged under additional seven-branes of the compactification. These seven-branes endow the low energy effective theory with additional, generically anomalous U(1) gauge group factors. The generalized Green-Schwarz mechanism cancels this anomaly, but the presence of the requisite coupling of the gauge field to an axion-like field in the four-dimensional effective theory generates a large mass for the gauge boson via the Stückelberg mechanism. Below the mass scale of this gauge boson, the theory will therefore retain an anomalous nearly exact global U(1) symmetry, which in appropriate circumstances can be identified with a U(1) Peccei-Quinn symmetry which we denote as $U(1)_{PQ}$. Let us note that this U(1) symmetry will appear as nearly exact to the four-dimensional effective field theory, and will be violated only by stringy instantons. It is therefore appropriate to associate the phase of X with the Goldstone mode associated with the breaking of this nearly exact global U(1) symmetry [3]. See subsection 3.1 and section 8 of [3] for further discussion.

The fields of the MSSM are charged under $U(1)_{PQ}$ with charges -2, $+1$ and -4 for the respective Higgs fields, chiral matter and X field of the F-theory GUT model. Integrating out the heavy $U(1)_{PQ}$ gauge fields also generates higher dimension operators in the low energy effective theory of the form [38]:

$$L_{\rm eff} \supset -\frac{4\pi\alpha_{\rm PQ}}{M^2_{{\rm U(1)}_{\rm PQ}}} e_X e_\Phi \cdot \int d^4\theta X^\dagger X \Phi^\dagger \Phi \tag{3.9}$$

where $M_{{\rm U(1)}_{\rm PQ}}$ denotes the mass of the heavy ${\rm U(1)}_{\rm PQ}$ gauge boson and the e's denote the respective charges of X and MSSM field Φ under ${\rm U(1)}_{\rm PQ}$, and $\alpha_{\rm PQ}$ is the fine structure constant of this gauge theory. Once X develops a vev as in equation (3.2), this generates an additional contribution to the masses squared of the scalar component of Φ at the messenger scale:

$$m^2_{\Phi,soft} = m^2_{\Phi,MGMSB} + \frac{e_\Phi \cdot e_X}{|e_X|} \cdot \Delta^2_{\rm PQ} \tag{3.10}$$

where we have introduced the PQ deformation parameter:

$$\Delta^2_{\rm PQ} \equiv 4\pi\alpha_{\rm PQ} \, |e_X| \left| \frac{F}{M_{{\rm U(1)}_{\rm PQ}}} \right|^2 . \tag{3.11}$$

In terms of the anomalous ${\rm U(1)}_{\rm PQ}$ gauge theory, this contribution can be interpreted as a supersymmetry breaking D-term. Insofar as the PQ deformation corrects the soft mass terms of the MSSM, the value of $\Delta_{\rm PQ}$ from prior considerations is on the order of $\sim$ 100 GeV. As we show later, this estimate is borne out by the cosmology of the F-theory GUT scenario.

By inspection of equation (3.10), the PQ deformation decreases the soft masses squared when $e_\Phi \cdot e_X < 0$. As a consequence, there is a limit to the size of $\Delta_{\rm PQ}$ before the PQ deformation induces a tachyon in the squark/slepton sector of the MSSM. The precise value of this vev depends on the value of Λ. For example, in a model with a single vector-like pair of messenger fields in the $5 \oplus \overline{5}$ of $SU(5)$, the minimal value of Λ consistent with the Higgs mass bound $m_{h^0} \geq 114.5$ GeV is $\Lambda \sim 1.3 \times 10^5$ GeV, and the maximal value of $\Delta_{\rm PQ}$ allowed before the mass squared of the lightest stau becomes tachyonic is $\Delta_{\rm PQ} \sim 290$ GeV. For vanishing PQ deformation, a bino-like lightest neutralino is the NLSP. On the other hand, for large PQ deformation, the NLSP can instead correspond to the lightest stau. The PQ deformation also plays a significant role in the dynamics of the X field which we analyze in detail in subsection 3.2. This is particularly relevant for cosmological considerations because as we explain in subsection 3.1, the axion and gravitino are both closely tied to the dynamics of X which is in turn controlled by the value of $\Delta_{\rm PQ}$.

3.1 Axion supermultiplet

In the axion solution to the strong CP problem, an anomalous global ${\rm U(1)}_{\rm PQ}$ symmetry is spontaneously broken at an energy scale f_a. The associated Goldstone mode then corresponds to the axion field, which we denote by a. In a supersymmetric theory, a fits into a complete supermultiplet given by one additional real bosonic degree of freedom s, a fermionic component $\psi_\perp$, and an auxiliary field $F_\perp$, which we assemble into the chiral superfield:

$$\mathcal{A} = a + is + \sqrt{2}\theta\psi_\perp + \theta^2 F_\perp . \tag{3.12}$$

The field s corresponds to the "saxion" and the field $\psi_\perp$ corresponds to the "axino". By definition, $\mathcal{A}$ couples to the QCD superfield strength through the coupling:

$$L \supset \mathrm{Re} \int d^2\theta \frac{\mathcal{A}}{16\pi^2} Tr_{SU(3)} W^\alpha W_\alpha. \tag{3.13}$$

In this section we review and slightly extend the analysis of [3] by showing that the components of $\mathcal{A}$ are to leading order given by the components of the chiral superfield X. In addition to its role in PQ symmetry breaking, the X field also plays a key role in supersymmetry breaking. In the field theory limit, this leads to an exactly massless Goldstino mode in the low energy theory. Precisely because X is the primary source of supersymmetry breaking, a linear combination given predominantly by the fermionic component of the X superfield with smaller contributions from other fermionic modes corresponds to the Goldstino. Away from the strict field theory limit, the Goldstino is eaten by the gravitino via the super-Higgs mechanism. To leading order, the axino therefore corresponds to the longitudinal components of the gravitino.

3.1.1 $\mathrm{U}(1)_{\mathrm{PQ}}$ Goldstone mode

We now describe the Goldstone mode associated with the breaking of the anomalous $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry. As explained in [3], to leading order, $\arg X$ corresponds to the axion field. Strictly speaking, however, this identification is not completely accurate because the axion superfield corresponds to a linear combination of X with subleading contributions from other chiral multiplets. The reason for this can be traced back to the way in which the PQ gauge boson develops a mass.

In general terms, the anomalous $\mathrm{U}(1)_{\mathrm{PQ}}$ gauge theory consists of n chiral superfields X_i with charges q_i such that:

$$Q \equiv \sum_{i=1}^{n} q_i \neq 0. \tag{3.14}$$

The anomaly for the corresponding U(1) gauge boson is then canceled via the Green-Schwarz mechanism. This corresponds to introducing an additional axion-like superfield $\mathcal{C}$ such that the chiral superfield $e^{\mathcal{C}}$ has charge $-Q$. $\mathcal{C}$ couples to the PQ vector multiplet via the D-term:

$$L \supset \int d^4\theta K(\mathcal{C} + \mathcal{C}^\dagger - Q \cdot V_{\mathrm{PQ}}), \tag{3.15}$$

where here, K denotes an appropriate Kähler potential. The corresponding gauge field develops a mass via the Stückelberg mechanism, leaving a nearly exact global symmetry in the low energy effective theory. Once some combination of the X_i's develop a vev, this global $\mathrm{U}(1)_{\mathrm{PQ}}$ will be broken, leaving behind a Goldstone mode.

The axion supermultiplet is given by a linear combination of $\mathcal{C}$ and the associated "phases" of the X_i's. Another linear combination of these fields is eaten by the vector multiplet. Assuming a canonical normalization for all of the X_i's, the direction in field space fixed by the D-term potential of the PQ seven-brane gauge theory is:

$$\sum_{i=1}^{n} q_i |X_i|^2 - QK' = \xi_{\mathrm{flux}}, \tag{3.16}$$

where here ξ_{flux} denotes a flux induced FI parameter. The directions unfixed by the D-term potential are conveniently parameterized in terms of chiral superfields Θ_i defined as:

$$X_j = |x_j| \exp(i\Theta_j) \tag{3.17}$$

The linear combinations of Θ_j neutral under $\text{U}(1)_{\text{PQ}}$ correspond to possible flat directions not fixed by the D-term potential. For example, in the context of the Fayet-Polonyi model, the $\text{U}(1)_{\text{PQ}}$ invariant combination:

$$\widehat{X} \equiv q \cdot X \tag{3.18}$$

develops a supersymmetry and global PQ symmetry breaking vev. This vev is stabilized by contributions to the Kähler potential for X and q [3]. The bosonic component of $\widehat{\Theta}$ is given by:

$$\widehat{\Theta} = a + is + \dots \tag{3.19}$$

where a is the axion and s is the saxion. The mode a takes values in the interval $-\pi < a < \pi$.

3.1.2 Supersymmetry breaking and the Goldstino

In the above analysis, we assumed that supersymmetry was unbroken. At energy scales below $x \sim 10^{12}$ GeV, supersymmetry is broken via the Fayet-Polonyi model described in [3]. In the strict field theory limit, spontaneous supersymmetry breaking generates a massless fermion corresponding to the Goldstino mode. The explicit form of the Goldstino mode depends on the details of F- and D-term breaking of a particular model. Nevertheless, certain features of the Goldstino mode and how it couples to fields of a particular model are universal, and our analysis will for the most part only rely on these well known features. As reviewed, for example in [36], the explicit form of the Goldstino mode is given as a linear combination of the fermions λ_a and χ_i respectively from the vector and chiral multiplets:

$$\widetilde{G} = \frac{iD^a}{\sqrt{2}}\lambda_a + F^i \chi_i \tag{3.20}$$

where the effects of supersymmetry breaking are encoded in non-zero vevs for some subset of the D^a and F^i. Taking into account the fact that M_{PL} is not infinite, spontaneous supersymmetry breaking implies that the Goldstino mode is eaten by the gravitino. The mass of the gravitino is:

$$m_{3/2}^2 = \frac{1}{3M_{\text{PL}}^2}\left(\sum_i \left|F^i\right|^2 + \frac{1}{2}\sum_a |D^a|^2\right). \tag{3.21}$$

In the explicit Fayet-Polonyi model presented in [3], there will in general be contributions to both the F- and D-term components of the gravitino. In addition to contributions to the D-term potential, the Fayet-Polonyi model contains an instanton induced contribution to the superpotential of the form:

$$W = M_{\text{PQ}}^2 e^{\mathcal{C}} X \tag{3.22}$$

where the mass scale $M_{\rm PQ} \sim M_{\rm GUT}$ which as explained in [3] can in general be different from the mass of the PQ gauge boson. For this range of mass scales, it turns out that the vev of $e^{\mathcal{C}}$ can naturally attain the value $\sim 10^{-17}$, as required to achieve $F \sim 10^{17}\,\text{GeV}^2$. With the identification $e^C \propto X_{n+1}$, we can alternatively view this Fayet-Polonyi model as a Fayet model of supersymmetry breaking with superpotential:

$$W = mX_1X_{n+1}. \tag{3.23}$$

As explained in [3], the full sector requires a non-trivial Kähler potential for X_1 and X_{n+1}. The F-term components of the various superfields are therefore determined as:

$$\overline{F}_1 = -mx_{n+1} \sim 10^{17}\ \text{GeV}^2 \tag{3.24}$$

$$\overline{F}_{n+1} = -mx_1 = -mx_{n+1}\frac{x_1}{x_{n+1}} \sim 10^{13}\ \text{GeV}^2, \tag{3.25}$$

where in the above we have plugged in rough representative values of $x_{n+1} \sim M_{\text{U}(1)_{\rm PQ}} \sim M_{\rm GUT}$ and $x_1 \sim 10^{12}\,\text{GeV}$ consistent with the estimates obtained in [3].

In addition to these F-term contributions to supersymmetry breaking, as explained in [38], we should also expect a contribution from D-term breaking, which in the present class of models is given as:

$$D = -\frac{4\pi\alpha_{\rm PQ}}{M^2_{\text{U}(1)_{\rm PQ}}}\sum_i q_i\left|F_i\right|^2 \simeq -\Delta^2_{\rm PQ}, \tag{3.26}$$

which is far smaller than the F-term breaking components.

To summarize the discussion presented above, the gravitino is predominantly given by the fermionic component of the X field. Nevertheless, for certain purposes, it is important that the gravitino corresponds to a linear combination of the Goldstino which contains additional contributions from the fermionic components of $e^{\mathcal{C}}$. For example, in the low energy effective theory for the X field, integrating out the heavy $\text{U}(1)_{\rm PQ}$ gauge boson generates the higher dimension operator:

$$L_X \supset -\frac{4\pi\alpha_{\rm PQ}}{M^2_{\text{U}(1)}}\int d^4\theta X^\dagger XX^\dagger X. \tag{3.27}$$

Once x and F_X develop non-zero values, this would appear to induce a mass term for the gravitino of order $\left|x\Delta^2_{\rm PQ}/F_X\right|$. In a more complete analysis, the fermionic mode ψ_X mixes with the fermionic components associated with $e^{\mathcal{C}}$. That this must be the case follows from the fact that in the limit $M_{\rm PL} \to \infty$, the gravitino is exactly massless. Indeed, a naive analysis suggests that the fermionic component $\psi_{\mathcal{C}}$ has a mass term of order $\left|x_{n+1}\Delta^2_{\rm PQ}/F_X\right|$ which is significantly larger. For most purposes, however, this distinction will not play any crucial role in many of the cosmological considerations discussed here.

3.2 Axion supermultiplet interaction terms

For the purposes of cosmological considerations, it is also important to determine the couplings between the axion supermultiplet and the matter content of the rest of the F-theory GUT model. Indeed, these interaction terms determine the lifetime of the saxion, which can have important consequences if it is sufficiently long-lived. In particular, the decay of the saxion can reheat the universe. The specific details of this reheating depends on the dominant mode of decay, as well as the primary decay rates.

As derived in [3], the effective action for the X field in 4d $\mathcal{N}=1$ superspace contains the terms:[5]

$$L_X = \int d^4\theta X^\dagger X^\dagger + \mathrm{Re}\int d^2\theta \frac{C_W \log X}{32\pi} Tr W^\alpha W_\alpha - \int d^4\theta C_\Phi \left(\log |X|^2\right)^2 \Phi^\dagger\Phi$$
$$- \frac{4\pi\alpha_{\mathrm{PQ}} e_\Phi e_X}{M^2_{\mathrm{U(1)_{PQ}}}} \int d^4\theta \Phi^\dagger\Phi X^\dagger X - m^2_{\mathrm{sax}}|X - \langle X\rangle|^2 \tag{3.28}$$

where in the above, Φ is shorthand for any MSSM chiral superfield, W^α denotes any gauge superfield strength, the multiplicative factors C_W and C_Φ are determined by the gauge couplings constants of the MSSM, as well as the quadratic Casimirs for the representations of the messenger fields, and e_Φ and e_X denote the $\mathrm{U(1)_{PQ}}$ charges of Φ and X. Here, the specific form of the saxion mass squared term is fixed by details of the Kähler potential for X, and the axion-like field $\mathcal{C}$ which enters the Green-Schwarz mechanism [3]. The explicit coupling of the axion supermultiplet is then given by performing the substitution:

$$X \to xe^{i\mathcal{A}} + \theta^2 F \tag{3.29}$$

in the Lagrangian density L_X. Although the PQ deformation dependence of the saxion mass is subject to order one tunings depending on the Kähler potentials of X and $\mathcal{C}$ [3], to make our discussion more concrete, we shall estimate the resulting mass using the higher dimension operator:

$$- \frac{4\pi\alpha_{\mathrm{PQ}} e_X e_X}{M^2_{\mathrm{U(1)_{PQ}}}} \int d^4\theta X^\dagger X X^\dagger X. \tag{3.30}$$

Substituting in our expression from equation (3.29), we shall take as a rough estimate the saxion mass squared as:

$$m^2_{\mathrm{sax}} = 4\,|e_X|\,\Delta^2_{\mathrm{PQ}} \tag{3.31}$$

or:

$$m_{\mathrm{sax}} = 4\Delta_{\mathrm{PQ}}, \tag{3.32}$$

where in the above we have used the fact that $|e_X| = 4$.

In the present context, however, the most important interactions are those which determine the decay modes of the saxion. The dominant interaction terms originate from the self-interactions of the X field chiral supermultiplet, and the soft mass terms associated

[5]It is important to note that this effective action is only valid a temperatures far below the messenger scale. Indeed, in the context of models where the dilaton directly couples to the QCD field strength, high temperature effects can potentially destabilize the value of the dilaton. Here, this problem is avoided because the actual coupling is only present in the low energy effective theory. See [39, 40] for further details on destabilization of the dilaton in other contexts. Nevertheless, it would be interesting to study in greater detail the high temperature phase of this system.

with gauge mediation effects. Expanding the kinetic term $|\partial_\mu X|^2$ in terms of the axion multiplet yields the model independent coupling between the axion and saxion:

$$L_X \supset \left(f_a^2 + f_a \cdot s\right)\left(\frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{2}\left(\partial_\mu s\right)^2\right). \tag{3.33}$$

In particular, the decay rate $s \to aa$ is given by:

$$\Gamma_{s\to aa} \sim \frac{1}{64\pi}\frac{m_{\rm sax}^3}{f_a^2} = \frac{1}{\pi}\frac{\Delta_{\rm PQ}^3}{f_a^2}. \tag{3.34}$$

Further details on universal couplings of the axion supermultiplet can be found in [41].

Gauge mediation generates soft mass terms for the scalars and gauginos of the MSSM, and also induces additional interaction terms for the saxion. The essential point is that the soft mass terms for a gaugino, λ, and scalar, ϕ of the MSSM are given as:

$$L_{\rm soft} \supset \frac{1}{2}m_{\lambda,soft}(x)\lambda\lambda + h.c. - m_{\phi,soft}^2\left(x,\overline{x}\right)|\phi|^2 \tag{3.35}$$

where in the above,

$$m_{\lambda,soft}(x) \propto \frac{F}{x} \tag{3.36}$$

$$m_{\phi,soft}^2(x,\overline{x}) \propto \left|\frac{F}{x}\right|^2. \tag{3.37}$$

As noted, for example, in [32], the explicit x dependence of $m_{\rm soft}$ implies that in both cases, X, and therefore the saxion directly couples to λ and ϕ. Performing the substitution $x \mapsto x + X$, it now follows that X couples to these fields as:

$$L_{\rm soft} \supset -\frac{1}{2}m_{\lambda,soft}(x)\frac{X}{x}\lambda\lambda + m_{\phi,soft}^2\left(x,\overline{x}\right)\frac{X}{x}|\phi|^2 + h.c. \tag{3.38}$$

Assuming that a given decay is kinematically allowed, the decay rate of the saxion to MSSM particles is therefore:

$$\Gamma_{s\to MSSM} \sim \frac{1}{2\pi m_{\rm sax}}\left(\frac{m_{\rm soft}^2}{f_a}\right)^2 = \frac{1}{8\pi\Delta_{\rm PQ}}\left(\frac{m_{\rm soft}^2}{f_a}\right)^2, \tag{3.39}$$

which is to be compared with the decay $s \to aa$. The ratio of these decay rates is:

$$\frac{\Gamma_{s\to aa}}{\Gamma_{s\to MSSM}} \sim 8\left(\frac{\Delta_{\rm PQ}}{m_{\rm soft}}\right)^4. \tag{3.40}$$

Although a sharp upper bound is somewhat flexible, in order to avoid a tachyonic mass, it is necessary to assume, for example, that $\Delta_{\rm PQ} \lesssim m_{\rm soft} \sim 100 - 1000\,\text{GeV}$. Introducing the branching ratio to axions

$$B_{s\to aa} = \frac{\Gamma_{s\to aa}}{\Gamma_{\rm sax}}, \tag{3.41}$$

where $\Gamma_{\rm sax}$ denotes the total decay rate, we conclude that for typical values of $\Delta_{\rm PQ}$ and $m_{\rm soft}$, $B_{s\to aa}$ is in the range:

$$10^{-3} < B_{s\to aa} < 10^{-1}. \tag{3.42}$$

JHEP04(2010)054

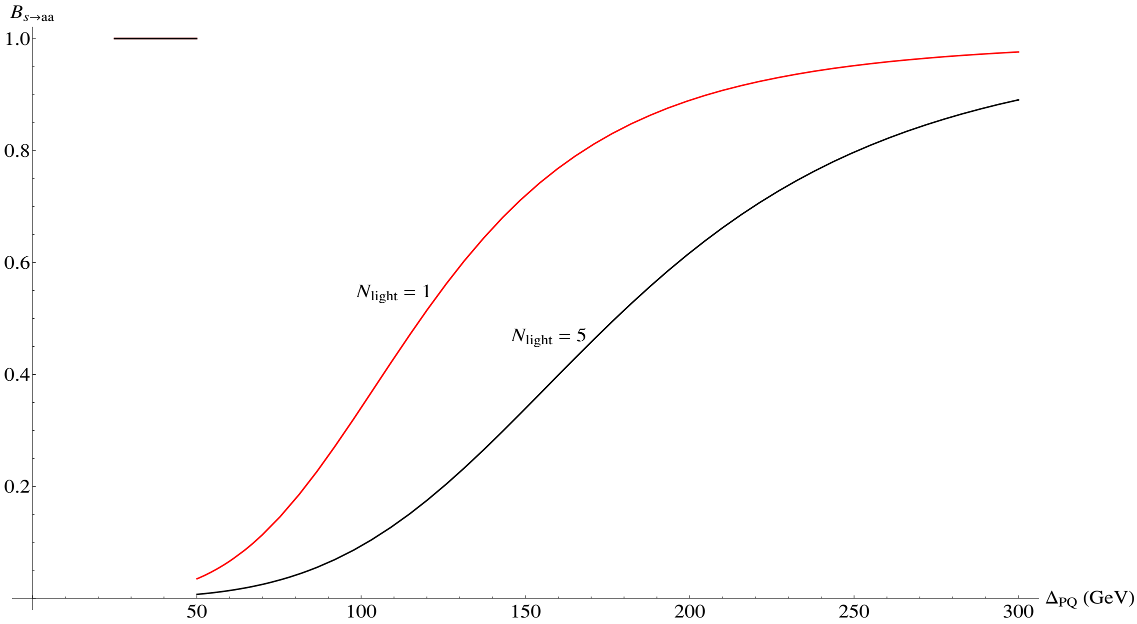

Figure 1. Plot of the "toy model" branching ratio of the saxion to axions and one (red) to five (black) species of MSSM fields as a function of Δ_{PQ} for fixed values of $f_a = 10^{12}\,\text{GeV}$ and $m_{\text{soft}} = 200\,\text{GeV}$. For the decay to a representative MSSM field, we have used the crude estimate provided by equation (3.39). This situation is somewhat idealized, because as Δ_{PQ} increases, the number of decay channels will increase, decreasing the overall branching ratio to axions.

See figures 1 and 2 for plots of the "toy model" branching ratios $\Gamma_{s\to aa}/(N_{\text{light}}\Gamma_{s\to MSSM} + \Gamma_{s\to aa})$ as functions of Δ_{PQ} with f_a and m_{soft} fixed to representative values. Here, N_{light} denotes the number of light MSSM species which are sufficiently light to allow a decay channel to an MSSM particle. In realistic models, N_{light} is a non-trivial function of Δ_{PQ}, although for illustrative purposes, we take it fixed in the figures.

When a decay to an MSSM particle is kinematically allowed, the branching fraction to MSSM particles will most likely dominate over decays to the axion. This is an important constraint in the context of decays to relativistic species, such as the axion. In a representative case, we can expect $\Delta_{\text{PQ}} \sim 100\,\text{GeV}$ and $m_{\text{soft}} \sim 10^{2.5}\,\text{GeV}$, in which case:

$$\frac{\Gamma_{s\to aa}}{\Gamma_{s\to MSSM}} \sim 10^{-1}. \tag{3.43}$$

Restricting to saxion decays to MSSM particles, we note that the primary decay channel through MSSM scalars proceed via the Higgs bosons and potentially the lightest stau $\widetilde{\tau}_1$ for sufficiently large PQ deformation. The saxion can also decay to MSSM fermions. Note, however, that in the supersymmetric limit, the axion supermultiplet coupling to the MSSM is far weaker. Because of this, our expectation is that the saxion couples more strongly to the fermionic superpartners in comparison to the fermions of the Standard Model. In the context of the MSSM, the gluinos are nearly 1000 GeV in mass. The primary decay channel through MSSM fermions therefore proceeds via decays to the two lightest neutralinos $\widetilde{\chi}_0^1$ and $\widetilde{\chi}_0^2$, which are primarily composed of the bino and wino. The respective masses of

JHEP04(2010)054

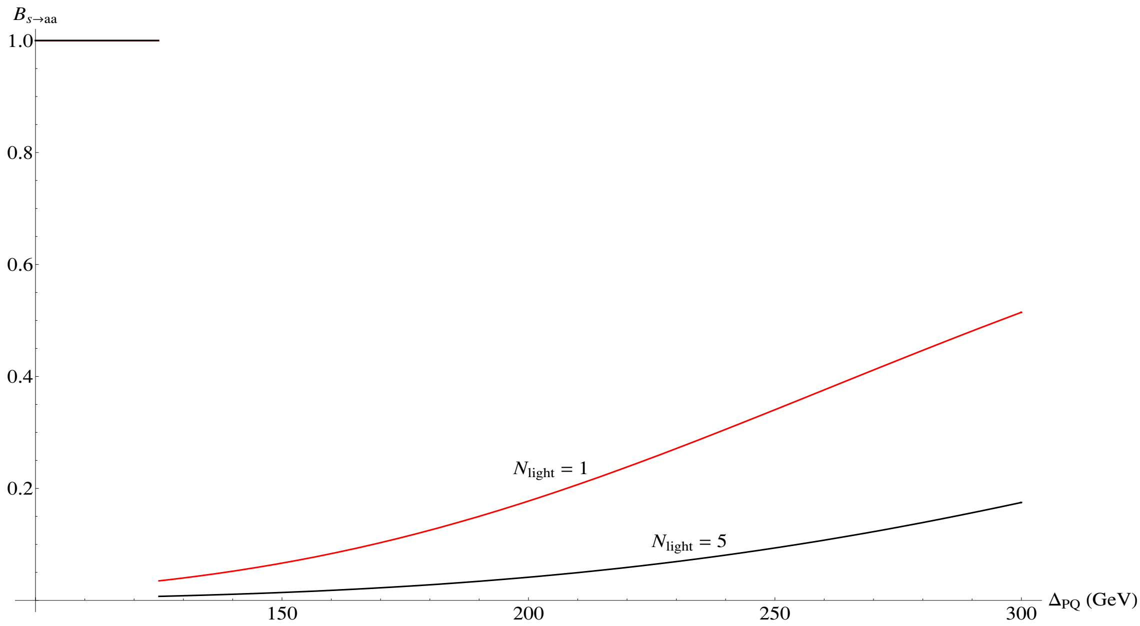

Figure 2. Plot of the branching ratio of the saxion to axions and one (red) to five (black) species of MSSM fields as a function of $\Delta_{\rm PQ}$ for fixed values of $f_a = 10^{12}\,\text{GeV}$ and $m_{\rm soft} = 500\,\text{GeV}$. For the decay to a representative MSSM field, we have used the crude estimate provided by equation (3.39). This situation is somewhat idealized, because as $\Delta_{\rm PQ}$ increases, the number of decay channels will increase, decreasing the overall branching ratio to axions.

these particles in the present class of models are $m_{\widetilde{\chi}^1_0} \sim 170\,\text{GeV}$ and $m_{\widetilde{\chi}^2_0} \sim 350\,\text{GeV}$. In all cases, the relative branching fractions are roughly determined by the relative masses of two species i and j to be:

$$\frac{\Gamma_{s\to xx}}{\Gamma_{s\to yy}} \sim \left|\frac{m^x_{\rm soft}}{m^y_{\rm soft}}\right|^4 . \tag{3.44}$$

Here it is important to note that the soft mass $m_{\rm soft}$ may in general differ from the actual mass of the particle. For example, whereas the Higgs field has mass $m_{h^o} \sim 115\,\text{GeV}$, the soft mass term which enters into the saxion decay rate formula is more accurately approximated by $m^{(0)}_{H_u} \sim 600\,\text{GeV}$ [3].

The saxion also couples to the gravitino. The model independent decay rate $s \to \psi_{3/2}\psi_{3/2}$ is given by:

$$\Gamma_{s\to\psi_{3/2}\psi_{3/2}} \sim \frac{1}{96\pi}\frac{m^3_{\rm sax}}{M^2_{\rm PL}}\left(\frac{m_{\rm sax}}{m_{3/2}}\right)^2 . \tag{3.45}$$

The relative branching fraction to gravitinos versus axions is therefore given as:

$$B_{s\to\psi_{3/2}\psi_{3/2}} = \frac{\Gamma_{s\to\psi_{3/2}\psi_{3/2}}}{\Gamma_{\rm sax}} = B_{s\to aa}\frac{\Gamma_{s\to\psi_{3/2}\psi_{3/2}}}{\Gamma_{s\to aa}} = \frac{2}{3}B_{s\to aa}\left(\frac{f_a}{M_{\rm PL}}\right)^2\left(\frac{m_{\rm sax}}{m_{3/2}}\right)^2 \tag{3.46}$$

$$\sim 2\times 10^{-4}\cdot B_{s\to aa}\left(\frac{\Delta_{\rm PQ}}{100\text{ GeV}}\right)^2\left(\frac{10\text{ MeV}}{m_{3/2}}\right)^2 . \tag{3.47}$$

JHEP04(2010)054

The decay rates obtained above allow us to determine the lifetime of the saxion. The lifetime of the saxion is given by the inverse of its decay rate:

$$\tau_{\text{sax}} = \Gamma_{\text{sax}}^{-1} = B_{s\to aa} \cdot \Gamma_{s\to aa}^{-1}, \tag{3.48}$$

where as before $B_{s\to aa}$ denotes the branching ratio of the saxion to axions. Combined with our expression for the decay rate to two axions given by equation (3.34), the lifetime of the saxion is roughly given by:

$$\tau_{\text{sax}} = B_{s\to aa}\Gamma_{s\to aa}^{-1} \sim B_{s\to aa}\frac{\pi f_a^2}{\Delta_{\text{PQ}}^3} \sim 2\times 10^{-6}\,\text{sec} \cdot B_{s\to aa}\left(\frac{100\text{ GeV}}{\Delta_{\text{PQ}}}\right)^3, \tag{3.49}$$

which shows that the saxion is long-lived.

Summarizing, the primary decay channel of the saxion is either to an MSSM field such as the Higgs, or to a pair of axions. In terms of the PQ deformation parameter Δ_{PQ}, the relevant decay rates are:

$$\Gamma_{s\to aa} \sim \frac{1}{\pi}\frac{\Delta_{\text{PQ}}^3}{f_a^2} \tag{3.50}$$

$$\Gamma_{s\to MSSM} \sim \frac{1}{8\pi\Delta_{\text{PQ}}}\left(\frac{m_{\text{soft}}^2}{f_a}\right)^2 \tag{3.51}$$

$$\Gamma_{s\to\psi_{3/2}\psi_{3/2}} \sim \frac{1024}{96\pi}\frac{\Delta_{\text{PQ}}^3}{M_{\text{PL}}^2}\left(\frac{\Delta_{\text{PQ}}}{m_{3/2}}\right)^2. \tag{3.52}$$

Having detailed the primary channels of the saxion, we now proceed to the cosmology of F-theory GUTs.

4 Cosmology of F-theory GUTs

A priori, a seemingly consistent particle physics model could be in severe conflict with cosmology. In this regard, cosmological considerations translate into constraints on properties of the particle physics model. In this section we study the cosmology of F-theory GUTs. Quite remarkably, we find that over the available range of parameters dictated by purely particle physics considerations, F-theory GUT scenarios appear to non-trivially satisfy many cosmological constraints. Further, we find that much of the tension typically present in gravitino cosmology finds a natural resolution in the context of F-theory GUTs, which has additional repercussions for issues related to generating a baryon asymmetry with a sufficiently high value of T_{RH}^0. Indeed, many popular mechanisms for generating sufficient baryon asymmetry hinge on allowing high values for the initial reheating temperature, which is commonly viewed as being in conflict with the requirements of gravitino cosmology. This resolution comes about because of a remarkable conspiracy in the mass of the gravitino and the expected dilution effects associated to the decay of the saxion. Thus, while one component of the axion supermultiplet might appear to create a source of tension, the other component completely relaxes it away.

JHEP04(2010)054

Cosmological constraints also provide an important window into UV sensitive features of the F-theory GUT model. Indeed, the PQ deformation is directly sensitive to the mass of the anomalous $U(1)_{PQ}$ gauge boson. This PQ deformation plays a prominent role in the dynamics of the saxion, which can in turn significantly impact the evolution of the Universe. In this way, constraints from cosmology on the dynamics of the saxion field directly translate into seemingly far removed constraints on the compactification and particle physics content of the F-theory GUT!

Because the ensuing discussion has many interlinked parts, we now summarize the main features which allow F-theory GUTs to evade the typically stringent bounds on the initial reheating temperature derived from bounds on the relic abundance of gravitinos. Starting from the initial temperature $T_{\rm RH}^{0}$, the Universe begins to cool. At high temperatures, the associated thermal bath converts MSSM particles into gravitinos. Due to the small total cross section of the gravitinos, these particles fall out of equilibrium at a relatively high temperature, $T_{3/2}^{f}$, which as before we will denote as the "freeze out" temperature. While all of this is occurring, however, the saxion modulus begins to oscillate at a temperature $T_{\rm osc}^{s}$. The essential point is that because its potential is so flat, the saxion will naturally be displaced away from its minimum. In fact, for values of the initial amplitude s_0 of the saxion field which are quite reasonable from the perspective of F-theory GUTs, the Universe eventually enters an era of saxion domination at a temperature $T_{\rm dom}^{s}$. At this point, the relic abundance of gravitinos is already determined and is given by the estimates already presented in section 2. The era of saxion domination continues until the saxion decays, at which point it reheats the Universe to a temperature $T_{\rm RH}^{s}$, releasing a large amount of entropy as a consequence of its decay. This dilutes the overall relic abundance of all species, such as gravitinos.

It remains to say whether the actual relic abundance of gravitinos is sufficiently low to remain in accord with overclosure bounds. As we will describe in greater detail in the sections to follow, although a priori, the oscillation temperature of the saxion and the gravitino mass are unrelated, in the context of F-theory GUTs, typical values for the saxion and gravitino mass lead to the relation:

$$\text{F-theory GUT} : T_{\rm osc}^{s} \sim T_{3/2}^{f}. \tag{4.1}$$

This turns out to have the remarkable consequence that *the overall relic abundance of gravitinos is independent of* $T_{3/2}^{f}$, $T_{\rm osc}^{s}$ and $T_{\rm RH}^{0}$! Moreover, the relic abundance of gravitinos thus obtained does not overclose the Universe, and could potentially correspond to a large component of the total dark matter. As reviewed in section 2, the axion can potentially also contribute to the overall dark matter content when it begins to oscillate at temperatures less than the reheating temperature of the saxion. This analysis only depends on the crude details of the saxion reheating temperature, which depending on the choice of inputs can sometimes be above, or below the axion temperature. As such, we will not dwell on this possibility in any great detail.

In addition, although far less likely, it is also possible to consider scenarios where the saxion does not dominate the energy density. In such cases, the usual very stringent overclosure bounds on the gravitino relic abundance apply, effectively requiring $T_{\rm RH}^{0}$ <

10^6 GeV for a gravitino of mass $m_{3/2} \sim 10-100$ MeV. This can occur, for example, when the initial amplitude of the saxion is far smaller, at around the scale set by the decay constant of the axion. Nevertheless, because this requires a significant calibration of various parameters such as the initial reheating temperature, in this paper we study the most natural situation where the saxion comes to dominate the energy density of the Universe.

While the saxion neatly resolves some puzzling features typically found in gravitino cosmology, it also has the potential to introduce additional issues. One such issue is that its decay products must not disrupt BBN. Indeed, as reviewed in section 2, the tight restrictions on the number of additional relativistic species which can be present translate into a constraint on the decay channels of the saxion. We find that this either requires introducing some additional weakly interacting particle into which the saxion can decay, or that the mass of the saxion must be sufficiently high so that other decay channels to MSSM particles become kinematically available.

Because the decay of the saxion indiscriminately dilutes various relics, it is important to check whether an appropriate baryon asymmetry can be generated in F-theory GUT models. Rather than presenting an impediment, the decay of the saxion appears to open up more possibilities for generating a suitable baryon asymmetry! This is due to two key features. First, because the relic abundance of gravitinos can quite naturally fall in a viable range, the usually very tight prohibition on increasing the initial reheating temperature T^0_{RH} is no longer present. Rather importantly, many mechanisms for generating a large baryon asymmetry require a high value for T^0_{RH}, which are commonly thought to be in conflict with the requirements of gravitino cosmology. Upon dispensing with the "gravitino problem", these options are once again available! Having said this, in typical models, the generated baryon asymmetry expected from such mechanisms is sometimes too large. In the present context, the same dilution effect discussed earlier turns out to retain an appropriate baryon asymmetry from scenarios such as standard leptogenesis in a natural range of parameters for F-theory GUT models.

The rest of this section is organized as follows. Because it will play a very central role in much of the analysis to follow, we begin by analyzing the history of the saxion, and determine the precise conditions necessary for saxion domination. After this, we study the relic abundance of the gravitino. To this end, we first frame the discussion in general terms, asking under which circumstances we can expect the decay of a cosmological modulus to render the gravitino relic abundance independent of the temperatures $T^f_{3/2}$, T^s_{osc} and T^0_{RH}. We find that there is a remarkably small range of parameters available, which are in fact consistent with the most naive expectations from F-theory GUTs! After this motivation, we present further details of how the saxion of F-theory GUTs satisfies all of these requirements. Next, we discuss the decay products generated by the decay of the saxion. After analyzing constraints from BBN, we show that standard leptogenesis can generate a suitable baryon asymmetry in the present class of models, with no fine tuning.

4.1 Cosmology of the F-theory GUT Saxion

Having outlined the general cosmology of F-theory GUTs, in this subsection we detail the primary role that the saxion plays as a cosmological modulus. In order for the saxion to dilute the relic abundance of a species such as the gravitino, it must be sufficiently long-lived in order for the coherent oscillations of the saxion field to dominate the energy density of the Universe. As obtained in section 3, the lifetime of the saxion is given by:

$$\tau_{\rm sax} \sim 2 \times 10^{-6}\,\text{sec} \cdot B_{s\to aa} \left(\frac{100 \text{ GeV}}{\Delta_{\rm PQ}} \right)^3 . \tag{4.2}$$

Due to its long lifetime, the initial amplitude of the saxion field and its subsequent coherent oscillation can potentially dominate over other contributions to the energy density of the Universe.

Because the saxion is a cosmological modulus, much of the general discussion reviewed in section 2 can now be applied to this special case of interest. Letting s_0 denote the initial amplitude of the saxion, the temperature at which the saxion begins to oscillate is given by equation (2.79) so that:

$$T^s_{\rm osc} \sim 0.3 \cdot \sqrt{m_{\rm sax} M_{\rm PL}} = 0.6 \cdot \sqrt{\Delta_{\rm PQ} M_{\rm PL}} \sim 10^{10} \text{ GeV} \cdot \left(\frac{\Delta_{\rm PQ}}{100 \text{ GeV}} \right)^{1/2} , \tag{4.3}$$

where in the second equality we have used the relation between the mass of the saxion and $\Delta_{\rm PQ}$ provided by equation (3.32). As anticipated, we note that this provides a link between cosmology and the PQ deformation.

Once the saxion begins to oscillate, it will continue to do so until it decays. Using the general formula for the decay of a modulus, the associated temperature of decay is given by:

$$T^s_{\rm RH} = T^s_{\rm decay} \sim 0.5 \cdot \sqrt{\Gamma_{\rm sax} M_{\rm PL}} \sim 0.4 \text{ GeV} \cdot B_{s\to aa}^{-1/2} \left(\frac{\Delta_{\rm PQ}}{100 \text{ GeV}} \right)^{3/2} , \tag{4.4}$$

where by a small abuse of notation, we have identified the decay temperature with a "reheating temperature", which is strictly speaking only correct if the saxion comes to dominate the energy density. Note in particular that $T^s_{\rm decay}$ is far lower than $T^s_{\rm osc}$. In addition, we also observe that the decay temperature of the saxion falls above the temperature required for BBN. Taking the maximal allowed branching ratio to axions consistent with BBN (which we will discuss later) so that $B_{s\to aa} \sim 1/6$, and for a representative value of $\Delta_{\rm PQ} \sim 100\,\text{GeV}$, we obtain the crude estimate:[6]

$$T^s_{\rm RH} \sim 1 \text{ GeV}. \tag{4.5}$$

In between the temperature at which it begins to oscillate and decay, the saxion may come to dominate the energy density of the Universe. Letting $T^s_{\rm dom}$ denote this temperature, this amounts to the condition:

$$T^s_{\rm osc} > T^s_{\rm dom} > T^s_{\rm decay}, \tag{4.6}$$

[6]Although the saxion is a cosmological modulus, it is interesting to note that in scenarios where it dominates the energy density of the Universe, it avoids the usual "moduli problem "because $f_a \ll M_{\rm GUT}$. Indeed, for a typical modulus of mass $m_\phi \sim m_{\rm sax}$, the analogous decay rate is of the form $\Gamma_\phi \sim m_\phi^3/\Lambda^2$, where Λ is of the GUT, or Planck scale. As a consequence, the corresponding reheating temperature would then be much lower, jeopardizing BBN.

where as reviewed in section 2 for a general cosmological modulus, the temperature $T^s_{\rm dom}$ is given by:

$$T^s_{\rm dom} \sim \frac{s_0^2}{M_{\rm PL}^2} \min(T^0_{\rm RH}, T^s_{\rm osc}). \tag{4.7}$$

There are a priori two natural scales associated to the initial amplitude of s_0. Because the saxion localizes on a matter curve with characteristic mass scale $M_X \sim 10^{15.5}$ GeV, it is natural to take:

$$s_0 \sim M_X \sim 10^{15.5} \text{ GeV}. \tag{4.8}$$

Comparing our expressions for $T^s_{\rm osc}$, $T^s_{\rm dom}$ and $T^s_{\rm decay}$, it follows that inequality (4.6) indeed holds for $s_0 \sim 10^{15.5}$ GeV. We note in passing that from the perspective of the effective field theory, it is also possible to consider smaller field ranges set by the value of the axion decay constant so that $s_0 \sim f_a \sim 10^{12}$ GeV. Note that in this case, $T^s_{\rm dom}$ is smaller than $T^s_{\rm decay}$, indicating that the saxion in this case never comes to dominate the energy density of the Universe.

Restricting to the most natural scenario where the saxion does indeed come to dominate the energy density, it will release a large amount of entropy when it decays. The associated dilution factor for any cosmological modulus again applies to the special case of the saxion, with the result:

$$D \sim \frac{M_{\rm PL}^2}{s_0^2} \frac{T^s_{\rm RH}}{\min(T^s_{\rm osc}, T^0_{\rm RH})}. \tag{4.9}$$

Treating separately the two cases $T^s_{\rm osc} > T^0_{\rm RH}$ and $T^s_{\rm osc} < T^0_{\rm RH}$, we therefore obtain the following expressions for the overall dilution factor:

$$D_{T^s_{\rm osc} > T^0_{\rm RH}} \sim 10^{-5} \cdot B_{s \to aa}^{-1/2} \left(\frac{10^{10} \text{ GeV}}{T^0_{\rm RH}}\right) \left(\frac{10^{15.5} \text{ GeV}}{s_0}\right)^2 \left(\frac{\Delta_{\rm PQ}}{100 \text{ GeV}}\right)^{3/2} \tag{4.10}$$

$$D_{T^s_{\rm osc} < T^0_{\rm RH}} \sim 10^{-5} \cdot B_{s \to aa}^{-1/2} \left(\frac{10^{15.5} \text{ GeV}}{s_0}\right)^2 \left(\frac{\Delta_{\rm PQ}}{100 \text{ GeV}}\right). \tag{4.11}$$

An important feature of these expressions is the overall dependence of the dilution factor on the initial reheating temperature, $T^0_{\rm RH}$. Indeed, we note that when $T^0_{\rm RH} < T^s_{\rm osc}$, the dilution factor becomes more potent as $T^0_{\rm RH}$ increases. This continues until $T^0_{\rm RH} \sim T^s_{\rm osc}$, at which point, the dilution factor ceases to depend on $T^0_{\rm RH}$. This situation closely parallels the $T^0_{\rm RH}$ dependence of the gravitino, to which we shall shortly turn.

Finally, it is also convenient to introduce the minimal reheating temperature required in order for saxion dilution to occur. This is given by the value of $T^0_{\rm RH}$ at which the dilution factor first equals one (the case of no dilution). Solving for $T^0_{\rm RH}$ in the equality:

$$1 = D_{\rm min} \sim \frac{M_{\rm PL}^2}{s_0^2} \frac{T^s_{\rm RH}}{T^0_{\rm RH}} \tag{4.12}$$

yields:

$$T^s_{D_{\rm min}} \equiv T^0_{\rm RH} \sim \frac{M_{\rm PL}^2}{s_0^2} T^s_{\rm RH}. \tag{4.13}$$

Using the explicit numerical values found in this section, it follows that the typical value of $T^s_{D_{\rm min}}$ is on the order of 10^5 GeV.

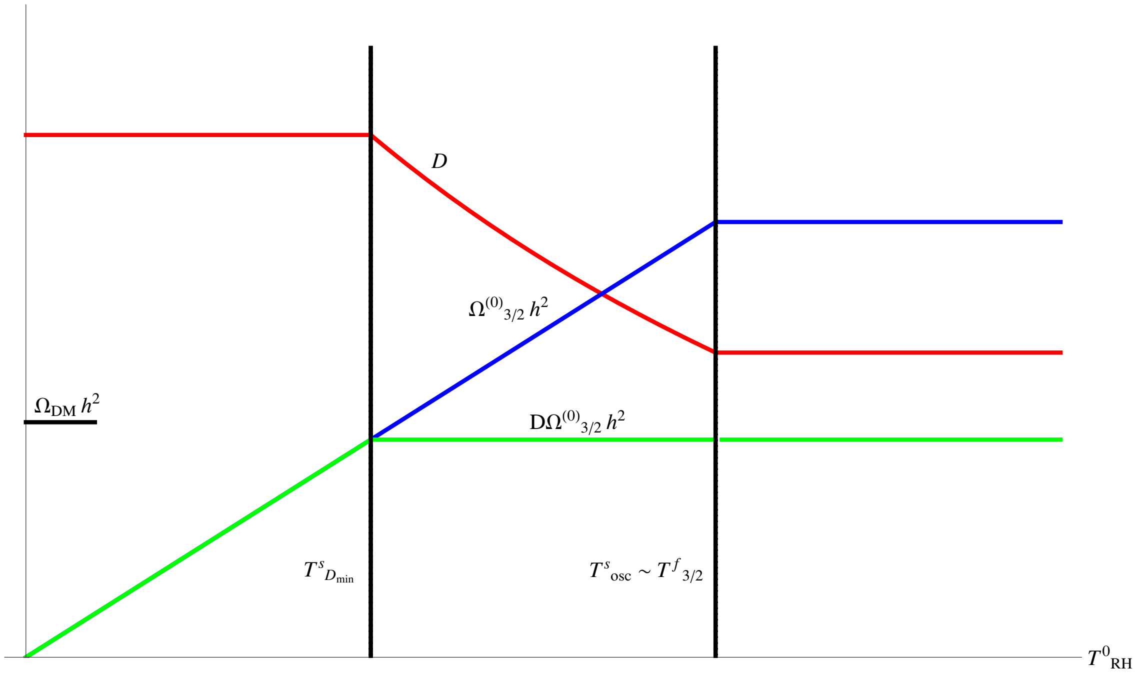

Figure 3. Schematic plot of the dilution factor D of the F-theory GUT saxion, the gravitino relic abundance in the absence of dilution, $\Omega^{(0)}_{3/2}h^2$ and the net relic abundance after taking account of the dilution factor of the saxion as a function of the initial reheating temperature $T^0_{\rm RH}$. The graph of the dilution is given with respect to a scale distinct from that for the relic abundances. The plot depicts the special case where the freeze out temperature for the gravitino $T^f_{3/2} \sim T^s_{\rm osc}$, the temperature at which the saxion begins to oscillate. When $T^0_{\rm RH}$ is below $T^s_{D_{\rm min}}$, there is no dilution factor ($D = 1$). In the special case where $T^f_{3/2} \sim T^s_{\rm osc}$, for all values of $T^0_{\rm RH} > T^s_{D_{\rm min}}$, the total relic abundance of gravitinos is independent of $T^0_{\rm RH}$. See figures 4 and 5 for schematic plots of scenarios where $T^f_{3/2} \neq T^s_{\rm osc}$.

4.2 The Saxion-gravitino connection

In the previous section we described the primary features of the saxion. The main point we found is that for typical values of F-theory GUT parameters, the oscillation of the saxion eventually comes to dominate the energy density of the Universe. The subsequent decay of the saxion will then dilute the relic abundance of all particle species. In this subsection we study the effects of this dilution on the relic abundance of gravitinos. We note that the idea of solving the gravitino problem due to a late decaying field has appeared for example in [42, 43]. In this regard, the quite natural way that these ideas automatically appear in F-theory GUTs provides strong motivation for this type of cosmological scenario.

Recall that in section 2 we reviewed the fact that for a gravitino of mass $m_{3/2} \sim 10 - 100\,\text{MeV}$, the resulting relic abundance would at first appear to violate the usual overclosure bound:

$$\Omega^T_{3/2}h^2 \leq 0.1. \tag{4.14}$$

We find that the dilution of the saxion naturally resolves this issue because of a remarkable

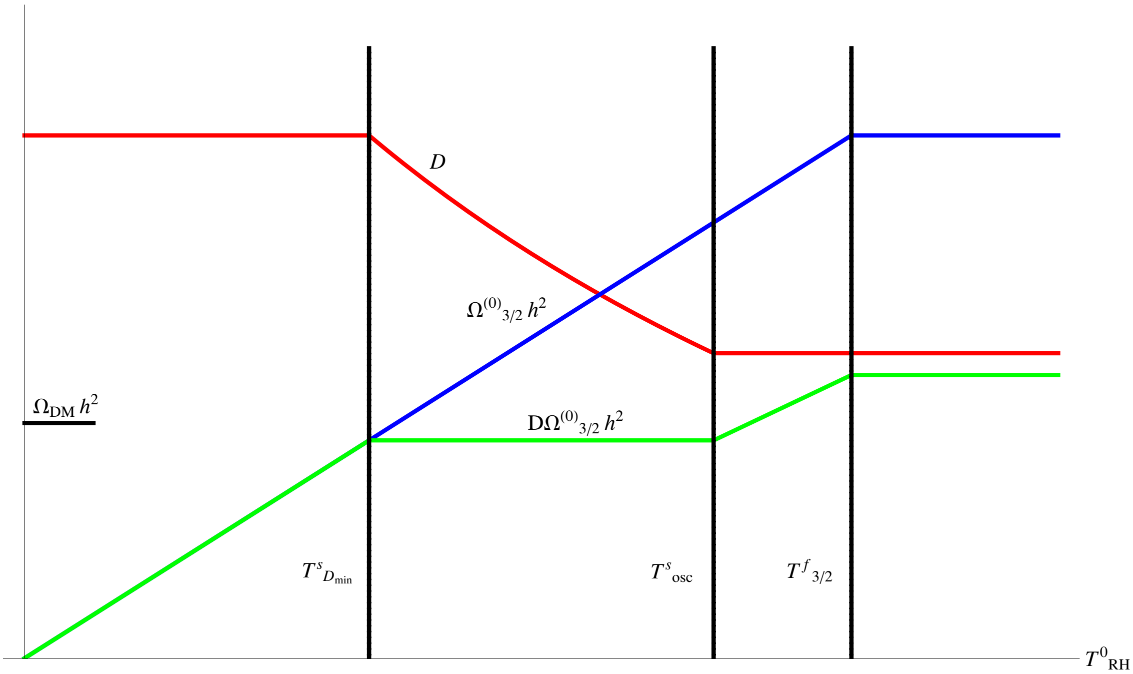

Figure 4. Schematic plot of the dilution factor D of the F-theory GUT saxion, the gravitino relic abundance in the absence of dilution, $\Omega^{(0)}_{3/2}h^2$ and the net relic abundance after taking account of the dilution factor of the saxion as a function of the initial reheating temperature $T^0_{\rm RH}$. The graph of the dilution is given with respect to a scale distinct from that for the relic abundances. The plot depicts the case where the freeze out temperature for the gravitino $T^f_{3/2} > T^s_{\rm osc}$, the temperature at which the saxion begins to oscillate. When $T^0_{\rm RH} < T^s_{D_{\rm min}}$, there is no dilution factor. Note that in this case, the total relic abundance of gravitinos increases for values of the initial reheating temperature such that $T^s_{\rm osc} < T^0_{\rm RH} < T^f_{3/2}$.

confluence of elements involving the masses of the gravitino and saxion, and the initial amplitude of the saxion. The basic point is that the relic abundance of gravitinos is governed by equation (2.72):

$$\Omega^T_{3/2}h^2 \sim D \cdot 2.7 \times 10^3 \cdot \left(\frac{\min(T^f_{3/2}, T^0_{\rm RH})}{10^{10}\ {\rm GeV}}\right)\left(\frac{10\ {\rm MeV}}{m_{3/2}}\right)\left(\frac{m_{\widetilde{g}}}{1\ {\rm TeV}}\right)^2 \tag{4.15}$$

where here, we have included explicitly the dilution factor associated with the decay of the saxion. In addition, the gravitino freeze out temperature is given by equation (2.62) as:

$$T^f_{3/2} \sim 2 \times 10^{10}\ {\rm GeV} \cdot \left(\frac{m_{3/2}}{10\ {\rm MeV}}\right)^2 \left(\frac{1\ {\rm TeV}}{m_{\widetilde{g}}}\right)^2 . \tag{4.16}$$

Note that the overall relic abundance is given by the minimum of the freeze out temperature $T^f_{3/2}$ and the initial reheating temperature $T^0_{\rm RH}$. Indeed, recall that for $m_{3/2} \sim 10\,{\rm MeV}$ and $m_{\widetilde{g}} \sim 1\,{\rm TeV}$, consistency with the overclosure bound would appear to require $T^0_{\rm RH} \lesssim 10^6$ GeV. Quite curiously, the amount of dilution associated with the saxion is also limited

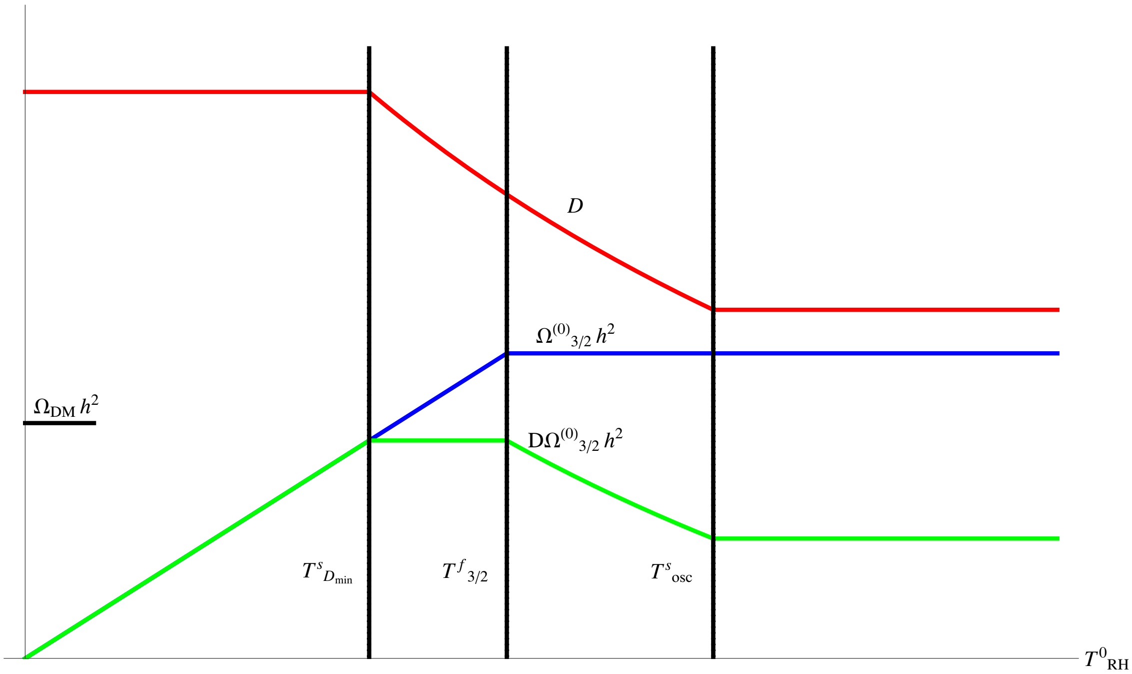

Figure 5. Schematic plot of the dilution factor D of the F-theory GUT saxion, the gravitino relic abundance in the absence of dilution, $\Omega^{(0)}_{3/2}h^2$ and the net relic abundance after taking account of the dilution factor of the saxion as a function of the initial reheating temperature $T^0_{\rm RH}$. The graph of the dilution is given with respect to a scale distinct from that for the relic abundances. The plot depicts the case where the freeze out temperature for the gravitino $T^f_{3/2} < T^s_{\rm osc}$, the temperature at which the saxion begins to oscillate. When $T^0_{\rm RH} < T^s_{D_{\rm min}}$, there is no dilution factor. Note that the total relic abundance of gravitinos decreases as $T^0_{\rm RH}$ increases in the range $T^f_{3/2} < T^0_{\rm RH} < T^s_{\rm osc}$.

by the minimum of its oscillation temperature, and $T^0_{\rm RH}$ so that:

$$D \sim \frac{M^2_{\rm PL}}{s_0^2}\frac{T^s_{\rm RH}}{\min(T^s_{\rm osc}, T^0_{\rm RH})}. \tag{4.17}$$

In other words, the thermally produced relic abundance of gravitinos is:

$$\Omega^T_{3/2}h^2 \sim 2.7\times 10^3 \cdot \frac{M^2_{\rm PL}}{s_0^2}\left(\frac{\min(T^f_{3/2}, T^0_{\rm RH})}{\min(T^s_{\rm osc}, T^0_{\rm RH})}\right)\left(\frac{T^s_{\rm RH}}{10^{10}\ \rm GeV}\right)\left(\frac{10\ \rm MeV}{m_{3/2}}\right)\left(\frac{m_{\tilde g}}{1\ \rm TeV}\right)^2. \tag{4.18}$$

While a priori, the oscillation temperature of the saxion and the gravitino mass are unrelated, in the context of F-theory GUTs, comparing the oscillation temperature $T^s_{\rm osc}$ of equation (4.3) with the freeze out temperature of the gravitino provided by equation (4.16), we have:

$$T^s_{\rm osc} \sim 10^{10}\ \text{GeV} \cdot \left(\frac{\Delta_{\rm PQ}}{100\ \rm GeV}\right)^{1/2} \tag{4.19}$$

$$T^f_{3/2} \sim 2\times 10^{10}\ \text{GeV} \cdot \left(\frac{m_{3/2}}{10\ \rm MeV}\right)^2\left(\frac{1\ \rm TeV}{m_{\tilde g}}\right)^2. \tag{4.20}$$

JHEP04(2010)054

In other words for typical values of $m_{3/2} \sim 10-100\,\mathrm{MeV}$, $m_{\widetilde{g}} \sim 1\,\mathrm{TeV}$ and $\Delta_{\mathrm{PQ}} \sim 100\,\mathrm{GeV}$, we obtain the remarkable relation:

$$T^s_{\mathrm{osc}} \sim T^f_{3/2}, \tag{4.21}$$

which should hold as an order of magnitude estimate. Returning to the gravitino relic abundance of equation (4.18), we therefore find:

$$\Omega^T_{3/2}h^2 \sim 0.1 \cdot \left(\frac{10^{15.5}\ \mathrm{GeV}}{s_0}\right)^2 \left(\frac{T^s_{\mathrm{RH}}}{1\ \mathrm{GeV}}\right) \left(\frac{10\ \mathrm{MeV}}{m_{3/2}}\right) \left(\frac{m_{\widetilde{g}}}{1\ \mathrm{TeV}}\right)^2 . \tag{4.22}$$

As a consequence, the overall relic abundance of gravitinos is independent of $T^f_{3/2}$, T^s_{osc} and T^0_{RH}!

Using our expression for T^s_{RH} given by equation (4.4) finally yields:

$$\Omega^T_{3/2}h^2 \sim 0.07 \cdot B^{-1/2}_{s\to aa} \left(\frac{10^{15.5}\ \mathrm{GeV}}{s_0}\right)^2 \left(\frac{\Delta_{\mathrm{PQ}}}{100\ \mathrm{GeV}}\right)^{3/2} \left(\frac{10\ \mathrm{MeV}}{m_{3/2}}\right) \left(\frac{m_{\widetilde{g}}}{1\ \mathrm{TeV}}\right)^2 , \tag{4.23}$$

which without any fine tuning satisfies the overclosure bound, and in certain circumstances could account for the observed dark matter density!

Turning the discussion around, the relation $T^s_{\mathrm{osc}} \sim T^f_{3/2}$ may be viewed as a type of "resonance condition" which preferentially selects a window of values for the mass of the gravitino. Setting $T^s_{\mathrm{osc}} \sim T^f_{3/2}$ and solving for $m_{3/2}$ in terms of Δ_{PQ}, we find:

$$m_{3/2} \sim 10\ \mathrm{MeV} \cdot \left(\frac{\Delta_{\mathrm{PQ}}}{100\ \mathrm{GeV}}\right)^{1/4} \left(\frac{m_{\widetilde{g}}}{1\ \mathrm{TeV}}\right) \tag{4.24}$$

as an order of magnitude estimate. An intriguing consequence of this formula is that the mass of the gravitino is fairly insensitive to the value of the PQ deformation parameter.

It is important to note that in the above estimate we have neglected potential temperature dependent corrections to the mass of the gravitino. In general, one would expect these corrections to decrease the mass of the gravitino above $\sqrt{F} \sim 10^{8.5}\,\mathrm{GeV}$. Our estimate for $T^f_{3/2} \sim 10^{10}\,\mathrm{GeV}$ is close to this scale, which suggests that there should be a mild decrease in the actual mass of the gravitino at this temperature. Thus, equation (4.24) should be viewed as a lower estimate for the gravitino mass, which is reassuring in that 10 MeV is on the lower end of the mass range for the gravitino in F-theory GUT models. As a consequence, the coincidence window is roughly in the range $m_{3/2} \sim 10-100\,\mathrm{MeV}$, which is intriguingly in the natural range expected for F-theory GUTs.

4.2.1 F-theory and a confluence of parameters

The result of the previous section suggests that some of the most distressing features of gravitino cosmology find a natural resolution in the context of F-theory GUTs. The crucial feature of this analysis is that although the gravitino relic abundance has non-trivial dependence on T^0_{RH}, this is exactly counterbalanced by the T^0_{RH} dependence of the dilution factor derived from the saxion.

Given this remarkable conspiracy, it is natural to ask whether more general models with a late decaying modulus ϕ could potentially exhibit similar properties. In fact, as we now explain, the required relations between the mass of this modulus, its associated decay width, and the mass of the gravitino are only satisfied in a small window of values.

The condition that the dilution factor exactly cancel the T^0_{RH} dependence of $\Omega^T_{3/2}h^2$ requires that the analogue of equation (4.21) must hold:

$$T^{\phi}_{\mathrm{osc}} \sim T^f_{3/2}. \tag{4.25}$$

Using the explicit expression for the oscillation temperature of a modulus as well as the value of $T^f_{3/2}$ given by equation (4.20), we obtain:

$$0.5\sqrt{m_\phi M_{\mathrm{PL}}} \sim T^{\phi}_{\mathrm{osc}} \sim T^f_{3/2} \sim 2\times 10^{10}\ \mathrm{GeV}\cdot\left(\frac{m_{3/2}}{10\ \mathrm{MeV}}\right)^2, \tag{4.26}$$

where for simplicity, we have set $m_{\widetilde{g}} \sim 1\,\mathrm{TeV}$. The mass of the gravitino is therefore given by:

$$\frac{m_{3/2}}{10\ \mathrm{MeV}} \sim 0.6\cdot\left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^{1/4}, \tag{4.27}$$

In other words, a cosmological modulus with mass on the order of $100-1000\,\mathrm{GeV}$ will lead to a gravitino abundance which is independent of $T^0_{\mathrm{RH}}, T^{\phi}_{\mathrm{osc}}$ and $T^f_{3/2}$ when the mass of the gravitino is in the range $10-100\,\mathrm{MeV}$!

We can also deduce further properties of the modulus by examining the overall relic abundance of gravitinos. Under conditions where equation (4.25) is satisfied, it is enough to consider the special case where $T^0_{\mathrm{RH}} > T^{\phi}_{\mathrm{osc}}, T^f_{3/2}$. In this case, the constraint $\Omega_{3/2}h^2 \leq 0.1$ translates to the condition

$$D_\phi\left(\frac{m_{3/2}}{2\ \mathrm{keV}}\right) \leq 0.1, \tag{4.28}$$

where in the above, D_ϕ denotes the dilution factor of the cosmological modulus. Using the explicit expression for D_ϕ reviewed in section 2, this becomes:

$$\Omega^T_{3/2}h^2 \sim \frac{M^2_{\mathrm{PL}}}{\phi_0^2}\sqrt{\frac{\Gamma_\phi}{m_\phi}}\left(\frac{m_{3/2}}{2\ \mathrm{keV}}\right) \leq 0.1, \tag{4.29}$$

where as before, Γ_ϕ denotes the decay rate of the modulus and ϕ_0 denotes the initial amplitude of this field. Parameterizing the decay rate as:

$$\Gamma_\phi \sim \frac{1}{64\pi}\frac{m^3_\phi}{\Lambda^2}, \tag{4.30}$$

where Λ is some characteristic scale associated with the dynamics of the ϕ modulus, we obtain the bound:

$$\Omega^T_{3/2}h^2 \sim 0.02\cdot\left(\frac{10^{15.5}\ \mathrm{GeV}}{\phi_0}\right)^2\left(\frac{10^{12}\ \mathrm{GeV}}{\Lambda}\right)\left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^{5/4} \leq 0.1, \tag{4.31}$$

which suggests a range of parameters similar to those of F-theory GUTs.

JHEP04(2010)054

4.3 Decay products of the Saxion

In the previous subsection we observed that the decay of the saxion and the associated release of entropy modifies the expected relic abundance of gravitinos, finding a remarkable confluence between the saxion oscillation temperature and the freeze out temperature for the gravitino. In addition we also found that the resulting relic abundance of thermally produced gravitinos is quite close to saturating the total amount of dark matter.

But while the decay of the saxion effectively dilutes previously generated thermal relics, the end products of its decay can re-introduce another source for these same particles! Due to their overall longevity, decays to gravitinos and axions comprise the main source of additional potential relics. In this subsection we compute the relic abundance from such "non-thermally produced" gravitinos and axions. Whereas the amount of non-thermally produced axions is entirely negligible, depending on the mass of the gravitino, non-thermal production of gravitinos can also contribute towards the total dark matter.[7]

For simplicity, in this subsection we restrict our attention to scenarios where the saxion comes to dominate the energy density of the Universe. Besides being the primary case of interest for F-theory GUTs, in the other more specialized case where the saxion is a subdominant component of the overall energy density, the resulting relic abundance will on general grounds generate a smaller component of the total matter content in comparison with a scenario with an era of saxion domination.

In a scenario where the saxion dominates the energy density of the Universe, the fraction of the energy density transferred to the i^{th} decay product is dictated by the branching ratio:

$$B_{s\to ii} = \frac{\Gamma_{s\to ii}}{\Gamma_{\text{sax}}} \tag{4.32}$$

where in the above $\Gamma_{s\to ii}$ the decay rate of the saxion to the i^{th} species, and as before, Γ_{sax} denotes the total decay rate of the saxion. In this case, the overall yield of the i^{th} species is given by:

$$Y_i^{\text{NT}} = \frac{n^{\text{NT}}_{i,after}}{s_{\text{after}}} = B_{s\to ii}\cdot\frac{s_{\text{before}}}{s_{\text{after}}}\frac{n_{sax,before}}{s_{\text{before}}} = \frac{3}{2}B_{s\to ii}\cdot\frac{T^s_{\text{RH}}}{m_s}. \tag{4.33}$$

where here, "before" and "after" are in reference to times close to the decay of the saxion. Note that as usual, Y_i is constant as the Universe subsequently evolves. The non-thermally produced relic abundance from the i^{th} species is therefore given by:

$$\Omega_i^{\text{NT}}h^2 = \left(\frac{s_0}{\rho_{c,0}}h^2\right)\cdot m_i Y_i^{\text{NT}} = \left(\frac{s_0}{\rho_{c,0}}h^2\right)\cdot\frac{3}{2}m_i B_{s\to ii}\frac{T^s_{\text{RH}}}{m_{\text{sax}}}. \tag{4.34}$$

We now compute the value of $\Omega_i^{\text{NT}}h^2$ in terms of the branching ratio $B_{s\to aa}$. Using the explicit expression for $B_{s\to\psi_{3/2}\psi_{3/2}}$ in terms of $B_{s\to aa}$ given by equation (3.46), plugging in the present values of $\rho_{c,0}$ and s_0 reviewed in section 2 as well as the value of T^s_{RH} obtained

[7] This is to be contrasted with for example, the result of [32], where in that case the production of gravitinos from the decay of the field responsible for supersymmetry breaking leads to essentially all of the gravitino relic abundance because the branching ratio to gravitinos is somewhat higher in the "sweet spot" scenario.

JHEP04(2010)054

in equation (4.4), the non-thermally produced relic abundance of gravitinos and axions are respectively given by:

$$\Omega^{\rm NT}_{3/2}h^2 \sim 0.9 \cdot B^{1/2}_{s\to aa}\left(\frac{10\ \text{MeV}}{m_{3/2}}\right)\left(\frac{\Delta_{\rm PQ}}{100\ \text{GeV}}\right)^{5/2} \tag{4.35}$$

$$\Omega^{\rm NT}_{\rm ax}h^2 \sim 5\times 10^{-9} \cdot B^{1/2}_{s\to aa}\left(\frac{m_a}{10^{-5}\ \text{eV}}\right)\left(\frac{\Delta_{\rm PQ}}{100\ \text{GeV}}\right)^{1/2}. \tag{4.36}$$

In the natural range of masses for F-theory GUTs, it follows that the relic abundance from axions is completely negligible. On the other hand, the relic abundance of non-thermally produced gravitinos can potentially play a more prominent role. For example, using the representative values $B \sim 10^{-2}$, $m_{3/2} \sim 20\,\text{MeV}$ and $\Delta_{\rm PQ} \sim 100\,\text{GeV}$, the resulting relic abundance of non-thermally produced gravitinos is ~ 0.05. On the other hand, when the mass of the gravitino is closer to the end range of F-theory GUT values at $\sim 100\,\text{MeV}$, the relic abundance is at most $\sim 10\%$ of the dark matter, so that such gravitinos could comprise an additional component of the dark matter.[8] This is to be contrasted with the case of the "sweet spot" model of supersymmetry breaking [32, 45], where the late decay of the field responsible for supersymmetry breaking generates most of the dark matter abundance.

4.4 BBN and F-theory GUTs

As reviewed in section 2, it is important to ascertain whether a given extension of the Standard Model disrupts the successful predictions of BBN. In this regard, we have already seen that the saxion can significantly alter the evolution of the Universe at temperatures above the start of BBN. In the most typical F-theory GUT scenario with an era of saxion domination, we find that the resulting reheating temperature is somewhat higher than $T_{\rm BBN}$. Indeed, this imposes only the mild constraint $\Delta_{\rm PQ} \gtrsim 1\,\text{GeV}$. A far more significant constraint from BBN originates from the bound on the number of relativistic species. In the context of models with minimal matter content, this translates into the condition that the saxion must be allowed to decay to additional species beyond the axion, which in turn imposes a lower bound on the mass of the saxion. In terms of the PQ deformation, this amounts to the condition:

$$\Delta_{\rm PQ} \gtrsim 60\ \text{GeV}. \tag{4.37}$$

Note that this bound is indeed in accord with the crude expectation that the value of $\Delta_{\rm PQ}$ is most naturally near the weak scale.

A late-decaying NLSP can also potentially alter the expected abundances of light elements generated by BBN. While a full study of BBN is beyond the scope of the present paper, a cursory inspection of the recent literature suggests that in comparison with standard cosmology, in a range of parameters for the gravitino and NLSP favored by F-theory

[8]Due to the fact that non-thermally produced gravitinos are created at temperatures only a few orders of magnitude different from BBN, it is interesting to ask whether the resulting relics are indeed "cold" or "warm" at the time of matter recombination. In principle, "hot" dark matter can disrupt structure formation. As dark matter candidates, both gravitinos and axions are typically both sufficiently non-relativistic at the time of matter recombination to constitute cold dark matter candidates. We refer the interested reader to [44] for further discussion on the distinctions between hot, warm and cold dark matter.

JHEP04(2010)054

GUTs, the expected abundance of light elements, such as the typically problematic ^7Li is in somewhat *better* accord with observation. This is due to the fact that the late decay of the NLSP in such scenarios can alter the reaction rates of BBN.

4.4.1 Decay channels of the Saxion

In subsection 2.7.1 we reviewed the general constraint on how many additional relativistic species can contribute such that the predictions of BBN remain in accord with observation. In this subsection we consider the special case associated with the decay products of the saxion, closely following the quite general analysis for saxion decays presented in [31]. There are in principle two possibilities, depending on whether or not the oscillation of the saxion comes to dominate the energy density of the Universe. As throughout, we shall specialize to the case where the saxion dominates the energy density of the Universe. Although the latter scenario is indeed more typical in the context of F-theory GUTs, for the sake completeness, in this subsection we study both possibilities in turn. Using the general constraint on the total number of allowed relativistic species reviewed in subsection 2.7.1, the contribution to the energy density from saxion generated axions at the time of BBN must satisfy the inequality:

$$\left(\frac{\rho_a}{\rho_r}\right)_{\text{BBN}} \leq \frac{7}{43}. \tag{4.38}$$

Assuming the saxion comes to dominate the energy density of the Universe, the energy density stored in the saxion is directly transferred into the background radiation, as well as the axions. As such, we obtain the estimate:

$$\left(\frac{\rho_a}{\rho_r}\right)_{\text{BBN}} \sim B_{s\to aa}, \tag{4.39}$$

with $B_{s\to aa}$ the branching ratio of the saxion to two axions. Inequality (4.38) therefore provides a bound on the branching fraction to axions:

$$B_{s\to aa} \leq \frac{7}{43}. \tag{4.40}$$

In the minimal F-theory GUT, the primary decay channel at small Δ_{PQ} is given by the saxion to two axions. In order to satisfy this bound, the F-theory GUT must include some additional mode into which the saxion can decay. While it is certainly possible to posit the existence of some additional mode beyond the ones already present in the MSSM, in a minimal scenario, the saxion will decay to MSSM degrees of freedom. As found in section 3, when available, the primary decay channel is for the saxion to decay to two Higgs bosons. This decay is kinematically allowed provided the mass of the saxion is sufficiently large:

$$m_{\text{sax}} \geq 2m_{h^0}. \tag{4.41}$$

Using the expected mass of the Higgs of $\sim 115\,\text{GeV}$, this translates into the following lower bound on the PQ deformation:

$$\Delta_{\text{PQ}} \gtrsim 60 \text{ GeV}. \tag{4.42}$$

We note that this crude kinematic constraint is fairly independent of the details of a particular F-theory GUT, such as the number of messengers, and in this regard is likely to be quite robust. Although it is also possible to derive a similar bound in the case where the saxion does not come to dominate the energy density of the Universe, in the context of F-theory GUTs, this is a far less likely scenario. We refer the interested reader to [31] for further details on this special case. We note that in general, the bound from the decay products of the saxion provides a somewhat tighter lower bound on $\Delta_{\rm PQ}$ in comparison to the bound obtained from the requirement $T^s_{\rm RH} > T_{\rm BBN}$.

4.4.2 Comments on the abundance of ^7Li

While the abundance of the light nuclei H^+, D^+, T^+, $^3He^{++}$, $^4He^{++}$ expected from BBN are all in reasonable accord with observation, there is also some tension between the abundance of ^7Li expected based on the Standard Cosmology, and the observed abundance, which is typically a factor of $0.2-0.5$ smaller. In fact, as mentioned in section 2, recent studies of the MSSM in scenarios with a gravitino in the mass range $10-100\,\text{MeV}$, with a bino or stau NLSP have recently been studied in [37] and even more recently in [35]. In this regard, it is interesting to note that the range of mass parameters expected in F-theory GUTs based on crude particle physics considerations are in rough accord with these studies. Although it is beyond the scope of this paper to address such detailed properties of BBN, it is quite encouraging that in [35], in the context of a gauge mediation scenario with a gravitino of mass $\sim 80\,\text{MeV}$ and a bino NLSP of mass $\sim 200\,\text{GeV}$ that the resulting abundance of ^7Li appears to be in better agreement with observation. It would be interesting to study this issue in greater detail.

4.5 Baryon asymmetry

In the previous subsection we explained that the additional particle content of F-theory GUTs does not appear to disrupt the reaction rates necessary in BBN. At a more basic level, however, it is important to verify that the primary input of BBN, namely a sufficient baryon number asymmetry:

$$\eta_B^{\rm obs} \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma} = \frac{s}{n_\gamma}\frac{n_B - n_{\overline{B}}}{s} \sim 7.04 \cdot Y_B \sim 6 \times 10^{-10}, \tag{4.43}$$

has in fact been generated! Here, Y_B denotes the net yield of baryons.

The creation of a sufficient baryon asymmetry is an especially acute problem in supersymmetric models with a gravitino LSP. Indeed, as reviewed in section 2, in models without an era of moduli domination, it is quite common to lower the initial reheating temperature $T^0_{\rm RH}$ to avoid overclosing the Universe from the thermal production of gravitinos. This in turn imposes strong constraints on the available mechanisms which can generate an appropriate baryon asymmetry. For example, in the context of models where the neutrinos of the Standard Model develop a small mass via the seesaw mechanism, heavy right-handed neutrinos couple to the Higgs up and lepton doublet through the superpotential term:

$$W \supset \lambda_\nu^{ij} H_u L^i N_R^j + M^i_{\rm maj} N_R^i N_R^i, \tag{4.44}$$

where in the above, $M_{\rm maj}$ denotes the Majorana mass of the right-handed neutrino, and λ_ν^{ij} is the Yukawa matrix in the neutrino sector, which for simplicity we shall take to be a 3×3 matrix. In the specific context of minimal F-theory GUTs which incorporate a seesaw mechanism, simple estimates for the mass of the lightest Majorana mass give [2]:

$$M_1 \sim 3\times 10^{12\pm 1.5}\ \text{GeV}. \tag{4.45}$$

In standard leptogenesis, the subsequent decay of the right-handed neutrino to the Higgs and lepton doublet generates an overall lepton number density which is converted via sphaleron processes to a baryon asymmetry. In order for this decay process to generate a sufficient baryon number asymmetry, however, the initial reheating temperature must be greater than the Majorana mass:

$$T^0_{\rm RH} \gtrsim M_{\rm maj}. \tag{4.46}$$

Indeed, for lower values of the initial reheating temperature, the decay products of the right-handed neutrinos are too dilute to generate the required baryon asymmetry.

But while the decay of the saxion dilutes the relic abundance of thermally produced gravitinos, it will indiscriminately also dilute any pre-existing baryon asymmetry! In this section we analyze whether a sufficient baryon asymmetry can be generated once the dilution effects of the saxion are taken into account, focussing on standard leptogenesis. We find that in the typical range of parameters for F-theory GUTs, standard leptogenesis typically generates a surplus baryon asymmetry which is diluted to acceptable values by the decay of the saxion. Similar studies on the compatibility of standard leptogenesis with a late decaying field which solves the gravitino problem have appeared, for example, in [42, 43]. Again, we find it very reassuring that F-theory GUTs provide a natural setting for realizing such scenarios.

Although we do not do so here, it is also possible to consider scenarios based on Dirac leptogenesis. In this case, an analogue of the seesaw mechanism generates small Dirac masses for the neutrinos, where the decay of the heavy particle associated with this "Dirac seesaw" generates light left- and right-handed neutrinos. Due to the difference in the efficiency of their interactions rates, this again can generate a lepton asymmetry, which is again converted to a baryon asymmetry. Insofar as standard leptogenesis can generate a viable level of baryon asymmetry, models with a similar range of parameters can also generate a sufficient baryon asymmetry in Dirac leptogenesis scenarios. While it would be interesting to study other alternative mechanisms for generating a large baryon asymmetry, it is beyond the scope of the present work to perform such an analysis. Indeed, the primary aim of this section is to demonstrate that in F-theory GUTs, standard mechanisms already generate an appropriate baryon asymmetry, without any additional assumptions.

4.5.1 Review of standard leptogenesis

We now briefly outline the main points of standard leptogenesis [46], following the review [47]. In extensions of the Standard Model which generate a suitable Majorana mass term for the light neutrinos via the seesaw mechanism, the decay of heavy right-handed neutrinos into leptons and Higgses can generate a lepton asymmetry. This process satisfies the Sakharov conditions reviewed in section 2 because by construction, the lepton number violation present in the Majorana mass term is converted to a baryon number violation via sphaleron processes. The violation of C and CP is somewhat more delicate, and at leading order originates from one loop contributions to the decay:

$$\nu_R \to l + h_u, \tag{4.47}$$

in the obvious notation. The necessity of the one loop contribution for C and CP violation can be established for example, by appealing to the optical theorem. The amount of CP violation can be characterized in terms of the parameter ϵ_1, which roughly measures the overall complex phase of this one loop contribution. As reviewed in [47], the net yield of leptons produced from this decay is:

$$Y_L = \frac{\kappa}{g_*}\,\epsilon_1, \tag{4.48}$$

where in the context of supersymmetric models, $g_*(MSSM) \sim 228.75$, and κ is the "washout" factor which quantifies to what extent the decay of heavy neutrinos occurs out of equilibrium. The washout factor is given by integrating the Boltzmann equations, and in the range relevant to us is given by [47]:

$$0 \lesssim r \lesssim 10 : \kappa\,(r) \sim \frac{1}{2\sqrt{r^2+9}} \tag{4.49}$$

$$10 \lesssim r \lesssim 10^6 : \kappa\,(r) \sim \frac{0.3}{r(\log r)^{0.8}} \tag{4.50}$$

$$10^6 \lesssim r : \kappa\,(r) \sim (0.1r)^{1/2} \cdot \exp\left(-\frac{4}{3}\,(0.1r)^{1/4}\right) \tag{4.51}$$

where in the above, r denotes a parameter associated with the efficiency of the reaction [47]:

$$r \equiv \frac{\Gamma_{N_1}}{H(M_1)} \sim \frac{g_*^{-1/2} \cdot (\lambda_\nu \lambda_\nu^\dagger)_{11}}{1.7 \cdot 32\pi}\,\frac{M_{\rm PL}}{M_1}, \tag{4.52}$$

with Γ_{N_1} the decay rate for the lightest of the heavy right-handed neutrinos. Sphaleron processes at high temperatures will automatically convert the lepton number asymmetry into a baryon asymmetry. The yield of net baryons Y_B from this conversion is:

$$Y_B = \frac{10}{31} \cdot Y_L = \frac{10}{31} \cdot \frac{\kappa}{g_*(MSSM)}\epsilon_1. \tag{4.53}$$

The amount of CP violation in a given model depends on the size of the hierarchy in the right-handed neutrino masses. For example, in the case of an "extreme" hierarchy, $M_2/M_1,\ M_3/M_1 \gtrsim 10^3$, the amount of CP violation is essentially dictated by a single complex number, and ϵ_1 satisfies the Davidson-Iberra bound [48, 49]:

$$\text{Extreme Hierarchical} : |\epsilon_1| \lesssim \frac{3\alpha}{16\pi}\,\frac{\delta m \cdot M_1}{\langle H_u \rangle^2} \equiv \epsilon_{\rm DI}, \tag{4.54}$$

where in the above, $\langle H_u \rangle$ denotes the vev of the Higgs up, and $\delta m \equiv m_{\rm max} - m_{\rm min} \sim 0.05\,\text{eV}$ is the mass splitting in the light-neutrino sector. Further, α is an order one parameter associated with the value of the Yukawa couplings in the neutrino sector. Finally, in the context of the large $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$ scenarios studied in [3], the actual value of $\langle H_u \rangle$ is given by:

$$\langle H_u \rangle = v \sin\beta \sim v \sim 246 \text{ GeV}. \tag{4.55}$$

Away from this extreme limit, the mass matrix for the right-handed neutrinos plays a more essential role, and there can be further sources of CP violation. For example, figure 1 of [50] illustrates that even when $M_2/M_1, M_3/M_1 \sim 10$, the value of ϵ_1 can deviate from ϵ_{DI} by one order of magnitude. More generally, in the case of less hierarchical masses, the resulting bound on ϵ_1 is [49, 50]:

$$\text{Less Hierarchical} : |\epsilon_1| \lesssim \max\left(\epsilon_{\text{DI}}, \frac{M_1^3}{M_2 M_3^2}\right) \sim O(1). \tag{4.56}$$

Plugging in all numerical factors, the resulting upper bound on the baryon asymmetry in these two situations is:

$$\text{Extreme Hierarchical} : \eta_B^{(0)} \lesssim 5 \times 10^{-7} \cdot \kappa \left(\frac{M_1}{10^{12} \text{ GeV}}\right) \tag{4.57}$$

$$\text{Less Hierarchical} : \eta_B^{(0)} \lesssim 10^{-2} \cdot \kappa, \tag{4.58}$$

where in the above, the superscript on the baryon asymmetry reflects the fact that the decay of a cosmological modulus could in principle dilute the total amount of baryon asymmetry generated. In general, we can expect an interpolation between the extreme, and less hierarchical scenarios. We now show that the expected range of Majorana masses in F-theory GUTs quite comfortably fits with the observed baryon asymmetry.

4.5.2 Saxion dilution and standard leptogenesis

In the previous subsection we reviewed the main features of standard leptogenesis, focussing in particular on the distinction between hierarchical and non-hierarchical Majorana masses. In this subsection we analyze the effect of saxion dilution on the net baryon asymmetry. As reviewed near equation (4.45), in F-theory GUTs where the right-handed neutrinos have large Majorana masses, the natural mass scale associated to such fields is roughly $\sim 3 \times 10^{12\pm1.5}$ GeV. In the present context then, standard leptogenesis is most natural in such cases when the initial reheating temperature T^0_{RH} is at or above this range of energy scales.

We now determine whether the dilution factor from the saxion decay is sufficiently small to avoid overclosure from gravitinos, whilst at the same time, sufficiently large to avoid completely diluting the necessary baryon asymmetry generated by standard leptogenesis. In the natural range of parameters for F-theory GUTs, the Majorana mass of the right-handed neutrinos $M_{\text{maj}} \gtrsim 3 \times 10^{12\pm1.5}$ GeV is greater than the freeze out temperature of the gravitino $T^f_{3/2} \sim 10^{10}$ GeV. As a consequence, the relic abundance of thermally produced gravitinos is given by:

$$\Omega^T_{3/2} h^2 \sim D \cdot \left(\frac{m_{3/2}}{2 \text{ keV}}\right) \leq 0.1. \tag{4.59}$$

In the range $m_{3/2} \sim 10 - 100\,\text{MeV}$, it follows that the dilution factor is bounded above by:

$$D \lesssim 2 \times 10^{-5} \cdot \left(\frac{10 \text{ MeV}}{m_{3/2}} \right). \tag{4.60}$$

Multiplying the baryon asymmetry estimated in equations (4.57) and (4.58) by the dilution factor, the baryon asymmetry is therefore bounded above by:

$$\text{Extreme Hierarchical}: \eta_B \lesssim 10^{-11} \cdot \kappa \left(\frac{M_1}{10^{12} \text{ Gev}} \right) \tag{4.61}$$

$$\text{Less Hierarchical}: \eta_B \lesssim 2 \times 10^{-7} \cdot \kappa, \tag{4.62}$$

where in the above estimate, we have used the fact that $m_{3/2} \gtrsim 10\,\text{MeV}$. To proceed further, we now estimate the overall size of the wash out factor, κ by determining the value of the parameter r in equation (4.52). Even without a complete theory of neutrino flavor, for our present purposes, it is enough to use the order of magnitude estimate for the Yukawa couplings:

$$(\lambda_\nu \lambda_\nu^\dagger)_{11} \sim \alpha_{\text{GUT}}^{3/2} \sim 8 \times 10^{-3} \tag{4.63}$$

Combined with the value of $g_*(MSSM) \sim 228.75$, the resulting value of r is:

$$r \sim \frac{6 \times 10^{12} \text{ GeV}}{M_1} \sim 2 \times 10^{\pm 1.5}, \tag{4.64}$$

where in the final estimate we have plugged in the explicit value of M_1 suggested by F-theory GUTs [2].

In the extreme hierarchical case, we see that a viable baryon asymmetry is only possible provided $M_1 > 10^{12}\,\text{GeV}$ and κ an order one parameter. Returning to equation (4.64), it follows that in this range r is indeed quite small, so that equation (4.49) implies $\kappa \sim 1/6$. Plugging this value of κ into (4.61), the baryon asymmetry is bounded above by:

$$\text{Extreme Hierarchical}: \eta_B \lesssim 2 \times 10^{-12} \cdot \left(\frac{M_1}{10^{12} \text{ GeV}} \right). \tag{4.65}$$

Generating the observed baryon asymmetry in this extreme case would then require:

$$\text{Extreme Hierarchical}: M_1 \sim 10^{14} \text{ GeV}, \tag{4.66}$$

which is remarkably close to the upper bound on M_1 expected in F-theory GUTs.

Next consider the more natural case for F-theory GUTs where the Majorana masses are not *extremely* hierarchical. In this case, inequality (4.62) is saturated provided $\kappa \sim 10^{-2} - 10^{-3}$. Returning to equations (4.49)–(4.51), this range of values requires κ to be in the second range so that $10 \leq r \leq 10^6$. Evaluating $\kappa\,(r)$ at some representative values, we have:

$$\kappa\,(r \sim 10) \sim 10^{-2} \tag{4.67}$$

$$\kappa\,(r \sim 100) \sim 10^{-3}. \tag{4.68}$$

In other words, in the range $10 \lesssim r \lesssim 100$, leptogenesis with less hierarchical masses generates the requisite baryon asymmetry. In terms of the Majorana mass M_1, this corresponds to the range:

$$\text{Less Hierarchical} : 10^{11} \text{ GeV} \lesssim M_1 \lesssim 10^{12} \text{ GeV}, \tag{4.69}$$

which is in the expected range of Majorana masses estimated in [2]!

To summarize, when the masses of the heavy neutrinos are not extremely hierarchical, we find that within a natural window of values for M_1, the resulting baryon asymmetry matches with the observed value. This is due to the interplay between the dilution due to the saxion, and the natural range of Majorana masses expected in F-theory GUTs. Indeed, F-theory GUTs elegantly reconcile the apparent tension between standard leptogenesis and the "gravitino problem".

4.6 Messenger relics

In the above sections, we have seen that the cosmology of F-theory GUTs is remarkably insensitive to the initial reheating temperature of the Universe, $T^0_{\rm RH}$. In the specific context of high scale gauge mediation scenarios, though, it is natural to ask whether the relic abundance of messengers will overclose the Universe. Indeed, if produced from the thermal bath, the large mass of these particles in F-theory GUTs, $M_{\rm mess} \sim 10^{12}$ GeV would lead to a very large relic abundance which even the decay of the saxion cannot dilute to an acceptable value. One possibility would be to take $T^0_{\rm RH} < M_{\rm mess} \sim 10^{12}$ GeV. In such a scenario the messengers would not have been produced by this initial condition assumption. However, this would not be attractive in our scenario, because we have seen that essentially all the relevant physics is independent of $T^0_{\rm RH}$. Moreover this upper bound on the value of $T^0_{\rm RH}$, is potentially in conflict with leptogenesis. A more natural assumption, in line with the spirit of the present paper is to assume that the messenger fields can decay to some lighter fields, such as those present in the MSSM. We are currently investigating explicit models of F-theory GUTs which take this feature into account [19].

5 Future directions

In this paper we have found that in F-theory GUTs, the gravitino and in some cases the axion can provide a prominent component of the total dark matter, which quite remarkably is independent of the initial reheating temperature of the Universe, $T^0_{\rm RH}$. On the other hand, such candidates leave open the issue of accounting for the recent experimental results such as PAMELA [51], which could potentially be explained in terms of dark matter physics. In this regard, it is important to investigate whether F-theory GUTs provide additional dark matter candidates [19].

In the context of F-theory GUTs, the decay of the saxion which dilutes the abundance of gravitinos will also dilute the relic abundance of any other dark matter candidate. Assuming that the dark matter originates from a cold thermal relic, its abundance scales inversely with its cross section:

$$\Omega_{\rm DM} h^2 \propto \frac{1}{\langle \sigma_{\rm DM} v_{\rm DM} \rangle}. \tag{5.1}$$

Letting M denote a characteristic mass scale associated with the dark matter, $\sigma_{\mathrm{DM}} \sim M^{-2}$, which would naturally suggest a mass scale of order $M \sim 1\,\mathrm{TeV}$. Taking into account the dilution factor $D \sim 10^{-4} - 10^{-5}$ from the decay of the saxion would instead suggest that $M \sim 100\,\mathrm{TeV}$. The downside to this is that this also lowers the cross section by a factor of $10^{-4} - 10^{-5}$. Unless there is a substantial enhancement either in its density or cross section at small velocities, this type of dark matter candidate would then be too small to be detectable in current dark matter searches.

Nevertheless, an exact analysis of potential dark matter candidates depends on the details of a given model. In this regard, the tight structure of F-theory GUTs also provides additional candidates associated with degrees of freedom located near the F-theory GUT seven-brane. For example, four-dimensional GUTs always include a $\mathrm{U}(1)_{B-L}$ gauge boson. We are currently investigating the details of such a scenario [19].

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JHEP04(2010)054

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