Homogenization, Thermal Fluctuations and the Landauer Bound in a Spin-1/2 Collision Model

Marco Pezzutto[1,2,†], **Mauro Paternostro**[2] and **Yasser Omar**[1]

[1]Physics of Information and Quantum Technologies Group, Instituto de Telecomunicações, Lisbon, Portugal and Instituto Superior Técnico, Universidade de Lisboa, Portugal

[2]Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queen's University, Belfast, United Kingdom

[†]m.pezzutto@qub.ac.uk

Abstract

The purpose of this work is to study the dynamics of a spin-1/2 particle in interaction via a collisional model with an environment constituted by a spin chain. The main focus lays in the investigation of the Landauer bound and of the asymptotic behaviour of the particle dynamics: whether the system reaches some stationary asymptotic state, whether homogenization to some specific environment preparation state occurs, and if so under which conditions. Implementing inter-environment interactions as well, the model allows for the study of memory effects and non-Markovianity.

Introduction: Collision models for open quantum system dynamics

The quantum mechanical description of individual closed systems has proven to be dramatically successful in a wide range of scenarios. In as many situations however, it fails to provide an accurate picture of the physical situation, since interactions with the surrounding environment cannot be ignored. The paradigm of *open quantum systems* therefore considers an object system S in interaction with some sort of environment E.

Collision models represent a theoretical tool for modelling explicitly various types of multipartite environments [1, 2]. $S-E$ interactions happen through *collisions*: sequential, "quick" interactions between the system and (a part of) the environment.

When there are no $E-E$ interactions between the environment subparts, the dynamics is fully *Markovian*. Otherwise if $E-E$ interactions are allowed, the dynamics can exhibit a non-Markovian character [3] characterized by memory effects and backflow of information from E to S.

Dynamical model

In our model we consider a spin-1/2 particle interacting with a "chain" of similar pariticles in an ordered sequence.

Let our system and environment particles all be governed by a spin-1/2 local Hamiltonian:

$$H_S = \frac{\hbar\omega_s}{2}\sigma_z, \qquad H_E = \frac{\hbar\omega_e}{2}\sigma_z, \qquad \omega_{s,e} > 0, \qquad H_0 = H_S + H_E$$

and suppose they interact pairwise via a Heisenberg Hamiltonian:

$$H_{\text{int}} = J\big(\sigma_x^S\sigma_x^E + \sigma_y^S\sigma_y^E + \sigma_z^S\sigma_z^E\big).$$

At each iteration, the system interacts with a **new, fresh environment particle**, which is then **discarded** afterwards.

All environment particles are initially in a thermal state $\rho_\beta^E = e^{-\beta H_E}/\text{Tr}(e^{-\beta H_E})$, with some *noise* introduced as gaussian fluctuations in the inverse temperature β.

The full dynamics is governed by $H = H_S + H_E + H_{\text{int}}$:

$$\rho_n^S \otimes \rho_\beta^E \quad \mapsto \quad \rho_{n+1}^{SE} = U(\rho_n^S \otimes \rho_\beta^E)U^\dagger, \quad U = \exp\Big(-\frac{i}{\hbar}H\tau_i\Big),$$

At each interaction, the *marginal* states ρ_{n+1}^S and ρ_{post}^E of the system and environment particles respectively are computed.

In can be shown that when all the environment particles are prepared in the *same* thermal state ρ_β^E, the dynamics allows for a **unique stady state**, the very same ρ_β^E: $[H, \rho_\beta^E \otimes \rho_\beta^E] = 0$.

Interenvironment interactions, when present, would occur alternatively to system-environment collisions, in exactly the same way.

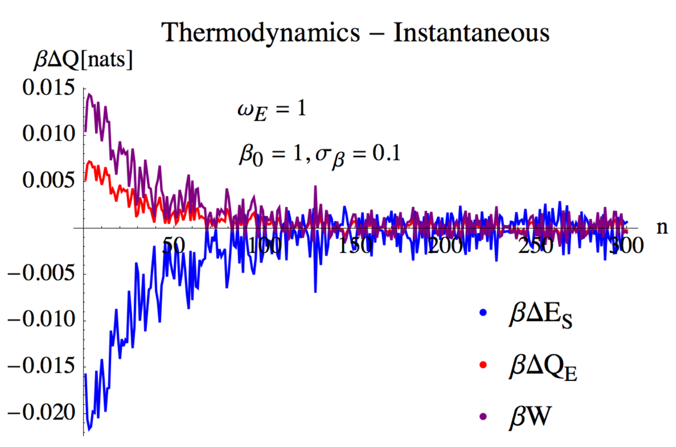
Thermodynamics

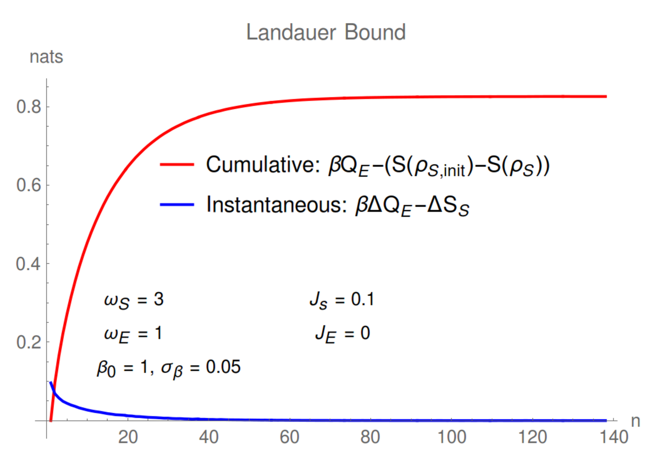
Given the marginal states, at each iteration step we can compute:

- the system *Von Neumann entropy*: $S_n = -\text{Tr}\big(\rho_n^S \log \rho_n^S\big), \quad \Delta S_n = S_n - S_{n-1}$
- the new system *Internal energy*: $\Delta E_n^S = \text{Tr}\big((\rho_n^S - \rho_{n-1}^S)H_S\big)$
- the *Heat Dissipated* into the environment: $\Delta Q_n^E = \text{Tr}\big((\rho_{\text{post}}^E - \rho_\beta^E)H_E\big)$
- the *Work* done by the system: $W_n = \Delta Q_n - \Delta E_n$

If the interaction does not conserve the local energies, i.e. $[H, H_0 + H_E] \neq 0$, then **work is done on/by the system**.

The following figures show two examples with a *non-interacting* environment:



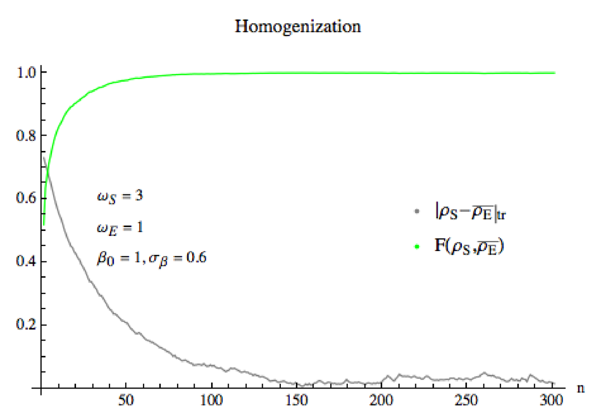


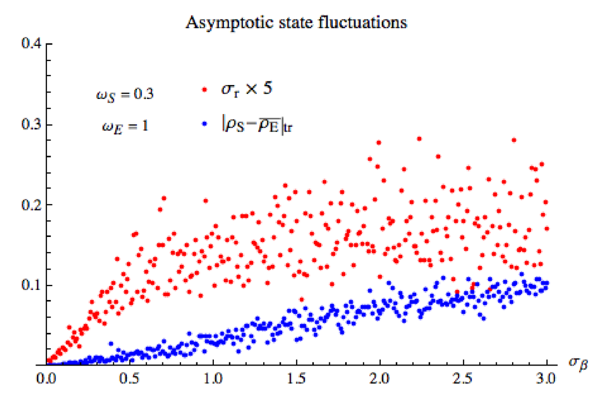
In addition, we investigate whether the **Landauer bound** $\Delta Q \geq \beta\Delta S$, recently also reformulated on a quantum statistical-mechanics ground [4, 5], is satisfied by the model.

Asymptotic behaviour - Markovian dynamics

For small fluctuations of β, the system *homogenizes* to the average environment state $\langle\rho_E\rangle$

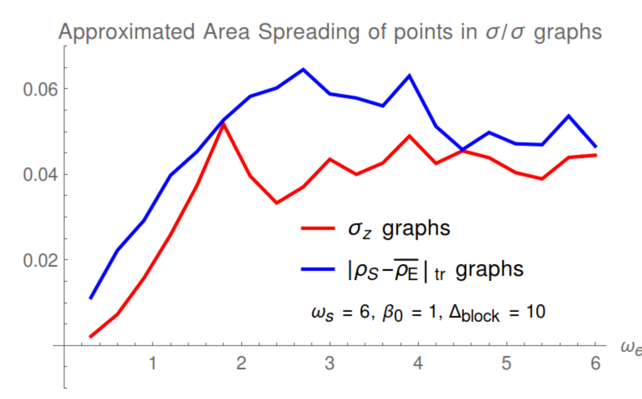
A "chaotic" behaviour emerges instead for large flucuations:

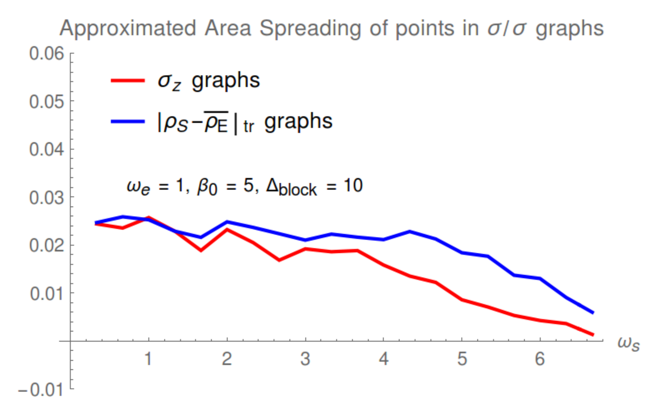




The asymptotic fluctuations are measured through the fluctuations in the system Bloch vector modulus and in the system state trace-distance from the average environment state.

We investigate further this "chaotic" behaviour by quantifying the *spread* in the cloud of points in the fluctuation/fluctuation graphs:





The "chaoticity" shows a peak against the environment frequency ω_e, whereas it is almost flat with that of the system ω_s.

Conclusions

In this work, we study numerically collision model for a spin-1/2 particle interacting with similar particles, with particular focus to the exploration of the asymptotic dynamics and of the thermodynamics in a noisy environment, in the Markovian regime.

A surprising character of "chaoticity" emerges in the asymptotic dynamics. We provided some hints of its dependency on the temperature and on system/environment frequencies. The process is however still unclear.

To which extent similar features also manifest in the non-Markovian case, and what other scenarios are opened by this dynamics, is the object of currently onging work.

References

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